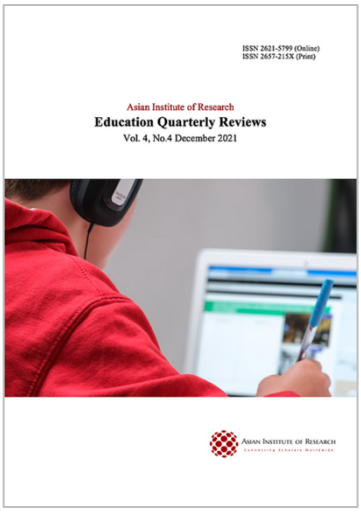

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Examining Problem-Solving and Problem-Posing Skills of Pre-Service Mathematics Teachers: A Qualitative Study

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Abstract
The aim of this study is to examine the problem-solving processes and problem-posing skills of pre-service mathematics teachers, which consists of four stages (understanding the problem, preparing a plan for the solution, applying the plan, evaluating) defined by Polya (1997) with the progressive scoring scale based on the alternative assessment approach. Qualitative research approach has been adopted in the study. Participants of the study consist of 71 pre-service teachers studying at the department of primary education mathematics teaching at the education faculty of a state university in the Southeastern Anatolia region of Turkey. Since the problem solving and problem posing behaviors of the participants were examined separately in the study, the gradual scoring scale developed by Baki (2008) was used. As a result of the analysis, it was determined that the participants showed the highest performance in the category of understanding the problem, and the lowest performance in the category of evaluation and problem posing. It was determined that participants who failed in the problem posing phase either wrote the same problem or could not write a problem. Another result reached in the study is that the participants had difficulties in expressing the operations in mathematical language.

Keywords: Mathematics Education, Problem-Solving, Problem-Posing, Evaluation, Pre-Service Mathematics Teachers

1. Introduction

Mathematics develops thinking, reasoning and problem solving skills that a person is likely to encounter in the real world (Bonotto & Santo, 2015). Problem solving, which is an integral part of mathematics, is not only an aim of learning mathematics, but also an important way of doing mathematics (National Council of Teachers of Mathematics [NCTM], 2000). A problem is an activity for which the solution is not immediately obvious (Jones, 2003). A math problem is a task presented to students in an instructional setting that poses a question that needs to be answered, but students do not have a ready-made procedure or strategy to answer (Lester Jr & Cai, 2016). Problem solving means participating in a task whose solution method is not known beforehand (NCTM, 2000). Problem solving is a more creative activity that involves testing, modifying, and refining the formulation of a possible hypothesis until it is possible to construct a formal proof of a set of activity theory (Tall, 2002). Problem solving is not just a product, but a process or method of combining information in a new (non-routine) way to

solve a problem (Gonzales, 1998). Problem solving is a skill that consists of finding out what needs to be done to get what you want, it is the main purpose of school mathematics and is a common task of students at all educational levels from primary school to university (Najdowski, 2017). It is pointed out that starting from preschool or kindergarten, students should be taught in a way that encourages problem solving as well as understanding the concepts and procedures of mathematics (Lester Jr & Cai, 2016). At the same time, problem solving is the ability to formulate, analyze and solve mathematical problems, as well as evaluate strategies, methods and results (Norqvist, 2016).

1.1. Problem Solving

Problem solving involves knowing when to apply a general mathematical result (such as a theorem or a built-in algorithm) to a particular situation to obtain the desired information (Mukhopadhyay & Greer, 2001). Problem solving is also defined as the process in which an individual tries to find a solution to a non-routine mathematical problem (Brahier, 2016). Problem solving provides students with the opportunity to use and apply their mathematical skills and knowledge (Jones, 2003). As students solve problems, they can use any approach they can think of, any knowledge they have learned or built on the spot, and justify their ideas in a way they feel persuasive (Lester Jr & Cai, 2016). By learning to solve problems in mathematics, students must gain confidence in their thinking, habits of persistence and curiosity, and in unusual situations that will serve well outside of mathematics class (NCTM, 2000).

Different problem solving stages are mentioned by mathematics education researchers. Although these stages serve similar purposes, there are some differences between them in terms of the number of stages. The most accepted is Polya's four-step process; understanding the problem, preparing the plan for the solution, applying the plan and evaluating the solution (Polya, 1997). In addition, Schoenfeld (1985) grouped this process into six stages; reading the problem, analyzing it, researching the methods, making a plan, implementing the plan and verifying the result. In the problem solving process, these stages may not be applied sequentially, and there may be back and forth between the stages. Because students may have different solutions for a given problem. On the other hand, Gonzales (1998) stated that problem posing comes after Polya's four-stage problem solving process and can be added to the problem solving process as the fifth step. In this context, students create a new problem by making use of the ready/existing solved problem, adding or changing the problem situation.

1.2. Problem Posing

Education systems support the rapidly changing priorities process and teaching-learning strategies are affected by this context. Problem posing as a learning and thinking practice plays an important role in this change (Singer, Ellerton, & Cai, 2015). Problem posing is a fundamental component of teaching and learning mathematics (Leavy & O'Shea, 2011; Stickles, 2011). Problem posing is a reformulation of a problem that arises in the process of generating a new problem or solving a problem (Silver, 1994). Problem posing is a series of transformations of the original problem, and each successive problem shows progress towards a solution and also provides opportunities for action to further expand the scope of the original problem (Cifarelli & Sevim, 2015). The ability of students to solve problems effectively by being aware of possible problems they may encounter in their real lives can be achieved with the problem posing skills they have acquired (Turhan, 2011). Problem posing activities were included in various learning areas of the mathematics curriculum (Ministry of National Education [MoNE], 2013; 2018). The inclusion of problem posing in the mathematics curriculum provides a valuable tool in teaching mathematics. Problem posing in meaningful contexts can provide insight into students' thinking and understanding, appeal to students' different levels of competence, emphasize mathematical structure and fundamental concepts, help teachers become more mathematically competent, and ultimately improve the problem-posing skills of both teachers and students. (English, 2020). Problem posing is not only a learning goal, but also a tool that can provide experiences for students to explore and create mathematical problems (Putra, Herman & Sumarmo, 2017). Such metacognitive processes form the basis of mathematical power and autonomy (Lowrie, 2002). Since this process also focuses on problem solving, it is

recommended to include problem-solving and problem-posing activities in mathematics teaching, and to work with different types of problems that can be solved with different strategies (Ev-Çimen, 2018).

Problem posing is included in the fifth stage of problem solving stages in the mathematics curriculum of Turkey (MoNE, 2013). Despite interest in integrating mathematical problem into teaching practices, the cognitive processes that occur when solvers generate their own problems remain relatively limited about instructional strategies that can effectively encourage productive problem posing, and the effectiveness of involving students in problem posing (Cai, Hwang, Jiang & Silber, 2015). There are many potential processes in problem posing, and this process may vary depending on the type of problem being addressed. If individuals are only going to pose problems that they have solved or can solve, problem posing can be considered as a measure of problem-solving competence (Stickles, 2011).

In this context, the aim of this study is to examine the problem-solving processes and problem-posing skills of pre-service teachers, which consists of four stages (understanding the problem, preparing a plan for the solution, applying the plan, evaluating) defined by Polya (1997) with the progressive scoring scale based on the alternative assessment approach. In line with this purpose, it is hoped that the data obtained will contribute to the development of pre-service teachers' problem-solving and problem-posing skills by detecting the mistakes made by the pre-service teachers in the problem-solving process and in the problems they have established, taking measures to eliminate these mistakes.

2. Method

2.1. Research Model

Qualitative research approach has been adopted in the study. Qualitative research is a preferred research approach in terms of benefiting from the experiences of the people doing research and understanding their feelings and thoughts (Ekiz, 2009). Based on this approach, the case study offers the opportunity to examine one or more situations or events in depth with a limited number of samples (Çepni, 2012). In this study, it was preferred to use this method since students' understanding of the problem in the problem solving process, planning, implementing the plan, evaluation stages and problem posing stage were examined in detail.

2.2. Research Group

Participants of the study consist of 71 pre-service teachers studying at the department of primary education mathematics teaching at the education faculty of a state university in the Southeastern Anatolia region of Turkey. Purposeful sampling method was used in determining the participants. Purposeful sampling allows for in-depth study of situations that are thought to have rich information (Yıldırım & Şimşek, 2021). Participants of the study took courses on problem solving and problem posing skills. Problem solving and problem posing courses were carried out for three weeks. Problem solving and posing activities were uploaded to the system by the first author and implemented within a certain period of time. The author is an expert in problem solving and posing, as he has done high-level events and projects on problem solving and posing. The names of the participants who participated in the study were kept confidential in accordance with the research ethics, and the names of the participants were given codes such as P1, P2, P3, P4… P71.

2.3. Data Collection Tool

In this study, five problems prepared by researchers using MoNE textbooks (Uçak, Emir, Kelek, Kutlu & Kahraman, 2019) were used as data collection tools. In order to ensure the content validity of these problems, two experts in mathematics education and two mathematics teachers were consulted in terms of content, level and language, and two problems were created with the necessary arrangements and reduced to three problems. A

pilot study was conducted with 10 students in order to see the reliability of the study and the usefulness of the data collection tool. At the end of the pilot study, it was decided that one lesson was sufficient and the problems were understandable by the students. Thus, the data collection tool consisted of three verbal problems. Problem activities are as follows:

1) A mathematics teacher said to two students who asked her age: "My age is 15 more than the sum of your ages, 18 times the difference. After 8 years, my age will be 22 times the difference of your ages." Find the ratio of the current age of the teacher to the sum of the current ages of the students.
2) The weights of ingredients A, B, C, D to be used in a meal and the percentage ratio of ingredient C by weight are given in the table below.

	A	B	C	D
Percentage Rate (%)			24	
Weight		540	480	360

Find what percentage of the food ingredient A is according to the given students.

3) A store organized a "Buy 3 Pay 2" campaign. According to the conditions of the campaign, a person who buys 3 products will pay the price of the 2 most expensive products. Buying 6 different products worth 140, 40, 120, 190, 80 and 30 liras, find out how many percent discount Furkan will buy the products thanks to this campaign?

A total of three problems were applied by the researchers during one class hour. The implementation of the problems was carried out online with the participants through the SIUZEM system. During the application process, detailed explanations were given to the students by the researcher and they were asked to solve the given problems by considering each stage. During the problem posing phase, the students were asked to pose another problem related to the subject of the given problem.

2.4. Analysis of Data

Since the problem solving and problem posing behaviors of the participants were examined separately in the study, the gradual scoring scale developed by Baki (2008) was used. The progressive scoring scale consists of five categories. Under these five categories, there are four criteria, 0, 1, 2 and 3 points, which define the highest and lowest performance of each criterion. The maximum score a participant can get from each criterion is 3 points, and the minimum score is 0. The maximum score a participant can get from a problem is a maximum of 15 points and a minimum of 0 points. The answers given by the participants to the three problems were scored according to the gradual scoring scale and the researchers scored between 3-2-1-0 for each category given in this scoring key. In this scoring, the scores that the students received at each stage were evaluated separately. Since the purpose of the study is to examine the performances and difficulties experienced by the participants during the problem solving and posing stages, the frequency and percentage values of each category and behavior are given. In addition, the data obtained were supported by sample participant responses. The details of the gradual scoring scale are given in Table 1.

Table 1: Progressive scoring scale

Categories	Criteria	Point
Understanding the Problem	Full understanding of the problem	3
	Understanding a part of the problem	2
	Inability to understand the problem	1
	Failure to make any effort to understand the problem	0
Pr ep are	Choosing a strategy that will lead to the appropriate solution	3

	Choosing only one part of the strategy that will help the solution	2
	Choosing an inappropriate strategy	1
	No strategy chosen	0
Implementing the Plan	Reaching the appropriate and correct solution	3
	Making a solution that is partially correct	2
	Making an appropriate and incorrect solution	1
	Failure to do any solution	0
Evaluation	Solving the problem and the new problem created according to this problem	3
	Logical validation of results	2
	Partial confirmation of results	1
	Not knowing how to verify the result	0
Posing a Problem	The problem posed is logical and solvable.	3
	A new problem is created by changing the values of the problem.	2
	A logic error has been made in the created problem and cannot be solved.	1
	Same problem written or no problem written	0

In order to ensure the reliability of the study, the method of agreement between raters was used. Miles and Huberman (1994) call similar codes as "Agreement" and dissimilar codes as "Disagreement" and suggest the formula Percent of Agreement = Consensus / (Agreement + Disagreement) * 100 for encoder reliability. In the study, the percentage of agreement of the codes obtained by the researchers was found to be 86%. A Miles-Huberman reliability formula value of .70 and above indicates that the scoring is consistent (Yıldırım & Şimşek, 2021). The percentage of agreement obtained in this study also shows that the scoring is consistent.

In the analysis of the obtained data, descriptive analysis technique was used. Descriptive analysis is summarized and interpreted according to predetermined themes (Yıldırım & Şimşek, 2021). In this study, it was deemed appropriate to use this technique, since the criteria in the gradual scoring scale were considered as a category.

3. Results

In this section, the analysis of the data obtained, the findings and comments obtained as a result of the analyzes are given.

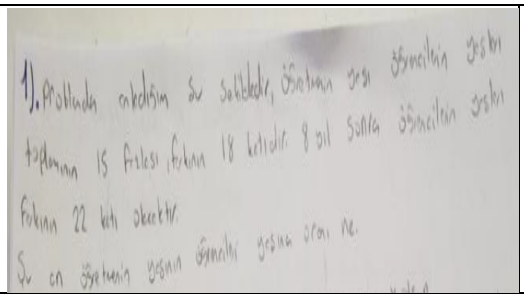
Table 2: Responses of Participants to the First Question

Categories	Criteria	f	%
Understanding the Problem	Full understanding of the problem	54	88
	Understanding a part of the problem	4	7
	Inability to understand the problem	1	2
	Failure to make any effort to understand the problem	2	3
Prepare the Plan	Choosing a strategy that will lead to the appropriate solution	54	88
	Choosing only one part of the strategy that will help the solution	5	8
	Choosing an inappropriate strategy	1	2
	No strategy chosen	1	2
Implementing the Plan	Reaching the appropriate and correct solution	54	88
	Making a solution that is partially correct	5	8

	Making an appropriate and incorrect solution	1	2
	Failure to do any solution	1	2
Evaluation	Solving the problem and the new problem created according to this problem	38	62
	Logical validation of results	6	8
	Partial confirmation of results	5	10
	Not knowing how to verify the result	12	20
Posing a Problem	The problem posed is logical and solvable.	35	57
	A new problem is created by changing the values of the problem.	6	10
	A logic error has been made in the created problem and cannot be solved.	0	0
	Same problem written or no problem written	20	33

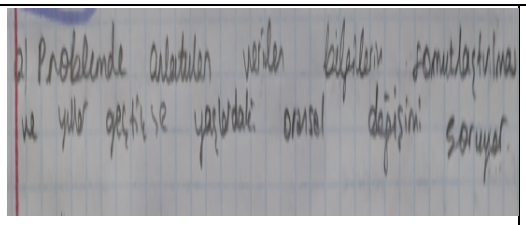
When Table 2 is examined, it is seen that the majority of the participants fully understood the first problem (88%). In addition, it was determined that 7% of the participants understood part of the problem, whereas the rate of participants who did not understand the problem and did not make any effort was 5%. When the plan preparation category related to the first problem was examined, it was determined that the majority of the participants (88%) developed an appropriate strategy, similar to the distribution in the category of understanding the problem. The rest of the participants either could not determine a correct strategy or did not make any effort to determine any strategy. When the plan implementation category of the first problem is examined, similar to the previous categories, the majority of the participants (88%) reached the appropriate and correct result, while the remaining participants could not reach an appropriate and correct result. When the evaluation category of the first problem was examined, it was determined that more than half of the participants (62%) were able to evaluate, but especially 30% had problems in making evaluations. Finally, in the problem-posing category, which has an important place in problem-solving skills, 57% of the participants set up logical and solvable problems. It was determined that 33% of the participants were unsuccessful in the problem posing process. First of all, the sample participant solution of the first problem, which received full points in the category of understanding the problem, is given in Table 3.

Table 3: P4 participant's response to the first problem in the category of understanding the problem

	What I understand from the problem is as follows: The age of the teacher is 15 times the sum of the ages of the students and 18 times the difference. After 8 years, they will be 22 times the age difference. What is the ratio of the age of the teacher to the age of the students at the moment?

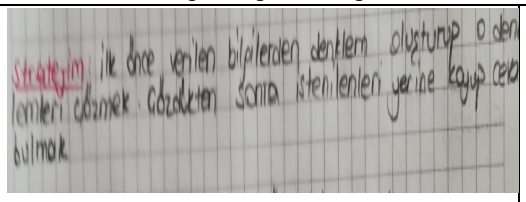
When Table 3 is examined, it is seen that P4 participant stated what is given and requested in the problem with his own sentence. Writing what is given and what is requested in the problem separately indicates that the problem is understood. The sample participant statement that was unsuccessful in the category of understanding the problem is shown in Table 4.

Table 4: P11 participant's response to the first problem in the category of understanding the problem

	Concretizing the information given in the problem and examining the proportional change in ages over the years.

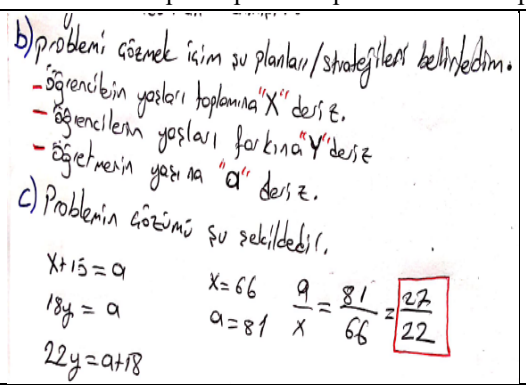
When Table 4 is examined, it is seen that the P11 participant did not understand the problem in the category of understanding the problem. The purpose of the problem is clear, and the participant expressed an opinion other than this purpose. There is no purpose to concretize the information. For the plan preparation category, some of the participants developed an incomplete or incorrect strategy. An example participant statement for this situation is given in Table 5.

Table 5: P7 participant's response to the first problem in the plan preparation category

	First, creating equations from the given information and solving those equations, after solving them, replace the desired ones and find the answer.

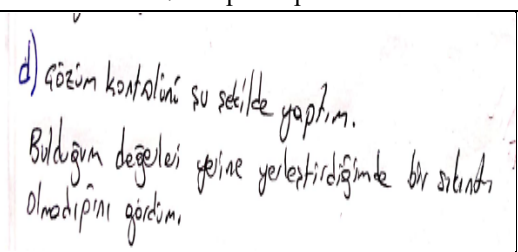
When Table 5 is examined, the plan developed by the P7 participant is included. When the plan was examined, it was stated as creating equations and solving these equations, but no information was given about any equation or how to solve the equations. In this case, the plan created does not have a concrete counterpart and can be considered as a failed plan. In the category of implementing the plan of the first problem, it was determined that some participants did not use the plans they stated or made mistakes in the implementation. An example participant statement for this situation is presented in Table 6.

Table 6: P1 participant's response to the first problem in the plan implementation category

	To solve the problem, I set the following plans: We call the sum of the ages of the students X We call the age difference of the students Y We call the teacher's age a The solution to the problem is as follows:

When Table 6 is examined, it is seen that P1 participant created a plan but applied this plan incorrectly. The mistakes he made while implementing the plan and his incomplete and incorrect transmission of the questions in the question caused him to fail in this category. An important detail here is the category of understanding the problem. If the understanding of the problem is incomplete or erroneous, then a systematic error occurs in the categories of plan development and implementation of this plan. It was determined that the same participant was unsuccessful in the evaluation category as well. The participant statement in question is presented in Table 7.

Table 7: P1 participant's answer to the first problem in the evaluation category

 d) Çözüm kontrolünü şu şekilde yaptım. Bulduğum değerleri yerine yerleştirdiğimde bir sıkıntı olmadığını gördüm.	I did the solution check as follows. When I replaced the values I found, I saw that there was no problem.

The reasons why the P1 participant failed in the plan creation and plan implementation categories were given. When Table 7 is examined, it is seen that the P1 participant was unsuccessful in the evaluation category as well. No checks were made and he claimed that his solution was correct. But when his solution is examined, the values he finds are wrong. As a result of checking the erroneous values, the error must be noticed. In this case, it can be said that the participant did not control any transaction. Finally, the responses of the participants belonging to the category of similar problem posing to the first problem were examined. It has been determined that some participants have difficulties in posing problems. It was determined that some participants made logical errors during the problem posing phase. An example participant statement for this situation is given in Table 8. In Table 9, the statement of the sample participant who successfully created a problem is given.

Table 8: P58 participant's response to the first problem in the problem posing category

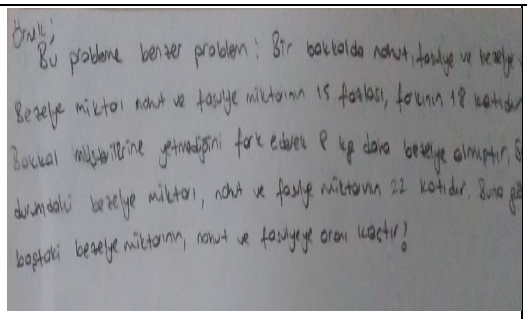
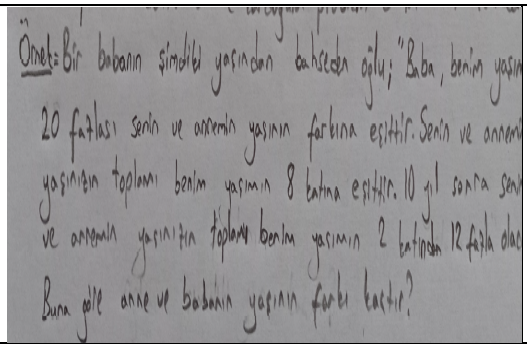
 Örnek: Bu probleme benzer problem: Bir bakkalda nohut, fasulye ve bezelye vardır. Bezelye miktarı nohut ve fasulye miktarının 15 katı, farkının 18 katıdır. Bakkal müşterilerine yetmediğini fark ederek 8 kg daha bezelye almıştır. Son durumdaki bezelye miktarı, nohut ve fasulye miktarının 22 katıdır. Buna göre başlardaki bezelye miktarının, nohut ve fasulyeye oranı kaçtır?	Problem similar to this problem: A grocery store has chickpeas, beans, and peas. The amount of peas is 15 times more than the amount of chickpeas and beans and 18 times the difference. The grocery store realized that it was not enough for its customers and bought 8 kg of peas. The amount of peas in the final state is 22 times the amount of chickpeas and beans. What is the ratio of the initial amount of peas to chickpeas and beans?

Table 9: P55 participant's response to the first problem in the problem posing category

 Örnek: Bir babanın şimdiki yaşından bahseden oğlu; "Baba, benim yaşımın 20 fazlası senin ve annemin yaşının farkına eşittir. Senin ve annemin yaşlarının toplamı benim yaşımın 8 katına eşittir. 10 yıl sonra senin ve annemin yaşlarının toplamı benim yaşımın 2 katından 12 fazla olacak. Buna göre anne ve babanın yaşının farkı kaçtır?	A father's son speaking of his present age; Dad, 20 more than my age is equal to the difference between your age and my mom's age. The sum of your and my mother's ages is equal to 8 times my age. In 10 years, the sum of your ages and my mother's ages will be 1 more than twice my age. What is the age difference between the parents?

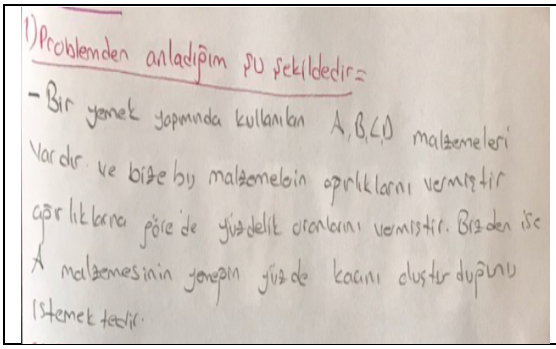
When Table 8 is examined, the problem posed by the P58 participant is not the same as the subject in the first problem. In the first problem, an age problem was given and it was emphasized that the age difference would not change after years, and an equation was asked to be established. However, these achievements were not included in the problem posed by the participant, and the participant failed in this category. Table 9 includes the problem of the P55 participant. The problem posed is quite original and parallel to the subject and outcome of the first problem.

Table 10: Responses of Participants to the Second Question

Categories	Criteria	f	%
Understanding the Problem	Full understanding of the problem	54	88
	Understanding a part of the problem	4	7
	Inability to understand the problem	1	2
	Failure to make any effort to understand the problem	2	3
Prepare the Plan	Choosing a strategy that will lead to the appropriate solution	55	90
	Choosing only one part of the strategy that will help the solution	4	6
	Choosing an inappropriate strategy	1	2
	No strategy chosen	1	2
Implementing the Plan	Reaching the appropriate and correct solution	53	87
	Making a solution that is partially correct	4	6
	Making an appropriate and incorrect solution	3	5
	Failure to do any solution	1	2
Evaluation	Solving the problem and the new problem created according to this problem	43	71
	Logical validation of results	5	8
	Partial confirmation of results	2	3
	Not knowing how to verify the result	11	18
Posing a Problem	The problem posed is logical and solvable.	35	57
	A new problem is created by changing the values of the problem.	7	12
	A logic error has been made in the created problem and cannot be solved.	0	0
	Same problem written or no problem written	19	31

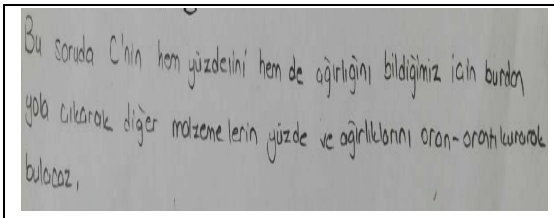
When Table 10 is examined, it is seen that the majority of the participants fully understood the second problem (88%). In addition, it was determined that 7% of the participants understood some of the problems, whereas the rate of participants who did not understand the problem and did not make any effort was 5%. When the plan preparation category related to the second problem was examined, it was determined that almost all of the participants (90%) developed a suitable strategy. The rest of the participants either could not determine a correct strategy or did not make any effort to determine any strategy. When the plan implementation category of the second problem is examined, similar to the previous categories, the majority of the participants (87%) reached the appropriate and correct result, while the remaining participants could not reach an appropriate and correct result. When the evaluation category of the second problem was examined, it was found that more than half of the participants (71%) were able to evaluate, but especially 21% had problems in making evaluations. Finally, in the problem posing category, which has an important place in problem solving skills, 57% of the participants posed logical and solvable problems. It was determined that 31% of the participants were unsuccessful in the problem posing process. First of all, the sample participant statement in the category of understanding the problem of the second problem is given in Table 11.

Table 11: P6 participant's response to the second problem in the category of understanding the problem

 1) Problemden anladığım şu şekildedir= - Bir yemek yapımında kullanılan A, B, C, D malzemeleri vardır. ve bize bu malzemelerin ağırlıklarını vermiştir ağırlıklarına göre de yüzdelik oranlarını vermiştir. Bizden ise A malzemesinin yemeğin yüzde kaçını oluşturduğunu istemektedir.	What I understand from the problem is as follows: There are A, B, C, D materials used in the making of a meal and the weights of these materials are given to us. Percentages are given according to the weights. On the other hand, we are asked what percentage of the meal A material constitutes.

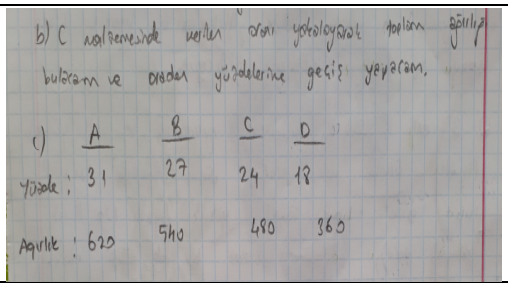
When Table 11 is examined, P6 participants did not write down what was given and requested in the problem separately, but presented a brief summary of the problem. As such, the understanding of the problem remains incomplete. A successful sample participant statement for the plan preparation category is presented in Table 12.

Table 12: P28 participant's response to the second problem in the plan preparation category

 Bu soruda C'nin hem yüzdesini hem de ağırlığını bildiğimiz için buradan yola çıkarak diğer malzemelerin yüzde ve ağırlıklarını oran-orantı kurarak bulacaz.	Since we know both the percentage and the weight of C in this question, we will find the percentage and weight of the other materials using ratio-proportionality, starting from here.

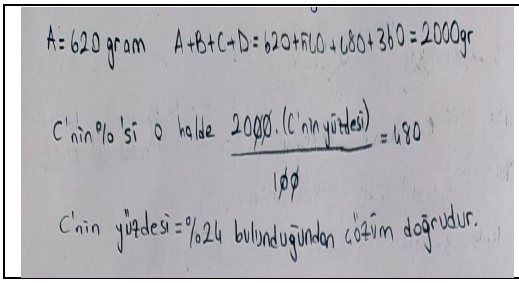
When Table 12 is examined, it can be said that P28 participant prepared a plan in accordance with the problem. In the category of implementing the plan of the second problem, it was determined that some participants did not use the plans they stated or made mistakes in the implementation. An example participant statement for this situation is presented in Table 13.

Table 13: P11 participant's response to the second problem in the plan implementation category

 b) C malzemesinde verilen oran yardımıyla toplam ağırlığı bulacam ve oradan yüzdelerine geçiş yapacam.	By catching the ratio given in material C, I will find the total weight and switch from there to the percentages.

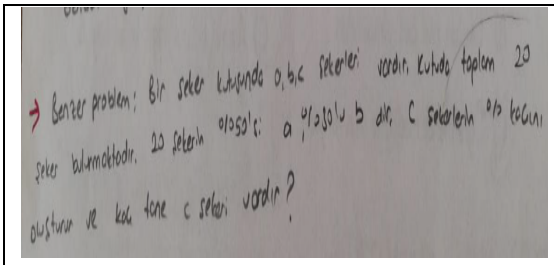
When Table 13 is examined, it is seen that P11 participant prepared a plan but did not take any action or comment during the implementation phase. The plan prepared by the P11 participant is also missing. This shortcoming has also manifested itself in the implementation phase. An example statement from the successful participants in the evaluation category of the second problem is given in Table 14.

Table 14: P33 participant's response to the second problem in the evaluation category

 A=620 gram A+B+C+D=620+540+480+360=2000gr C'nin %'si o halde 2000.(C'nin yüzdesi) / 100 = 480 C'nin yüzdesi=%24 bulunduğundan çözüm doğrudur.	A=620 Gram A+B+C+D=620+540+480+360=2000 gr The solution is correct since %C = 24%.

When Table 14 is examined, it is seen that the P33 participant made a successful evaluation. He wrote the result he found in place of the desired ones in the problem, found the percentage value and compared it with the value in the table. One of the most important points of being successful in the evaluation category is to understand the problem, prepare a plan to solve the problem and implement this plan correctly. It was determined that the P33 participant was successful in these categories, and it was determined that he was successful in the evaluation category as well. Finally, participant responses to the problem posing category of the second problem were examined. As in the first problem, it was determined that some participants had difficulties in posing problems in the second problem. In Table 15, the statement of the sample participant who was successful in the problem posing category is given.

Table 15: P47 participant's response to the first problem in the problem posing category

	Similar problem: There are a, b, c candies in a candy box. There are 20 candies in total in the box. Of 20 sugars, 50% is a and 30% is b. How many of the c sugars make up and how many c sugars are there?

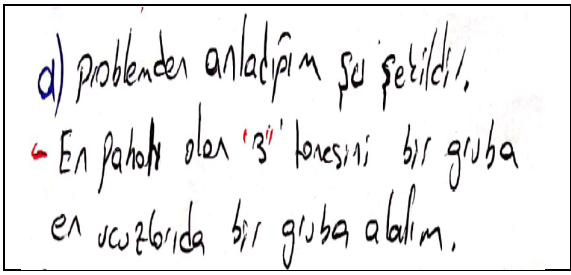
When Table 15 is examined, the problem posed by the P47 participant has similar gains with the desired gains in the second problem. Since the posed problem is a logical and solvable problem, it can be said that the participant is successful in this category.

Table 16: Responses of Participants to the Third Question

Categories	Criteria	f	%
Understanding the Problem	Full understanding of the problem	47	77
	Understanding a part of the problem	5	8
	Inability to understand the problem	1	2
	Failure to make any effort to understand the problem	8	13
Prepare the Plan	Choosing a strategy that will lead to the appropriate solution	45	74
	Choosing only one part of the strategy that will help the solution	5	8
	Choosing an inappropriate strategy	3	5
	No strategy chosen	8	13
Implementing the Plan	Reaching the appropriate and correct solution	40	66
	Making a solution that is partially correct	7	11
	Making an appropriate and incorrect solution	6	10
	Failure to do any solution	8	13
Evaluation	Solving the problem and the new problem created according to this problem	35	57
	Logical validation of results	4	7
	Partial confirmation of results	6	10
	Not knowing how to verify the result	16	26
Posing a Problem	The problem posed is logical and solvable.	27	44
	A new problem is created by changing the values of the problem.	4	7
	A logic error has been made in the created problem and cannot be solved.	1	1
	Same problem written or no problem written	29	48

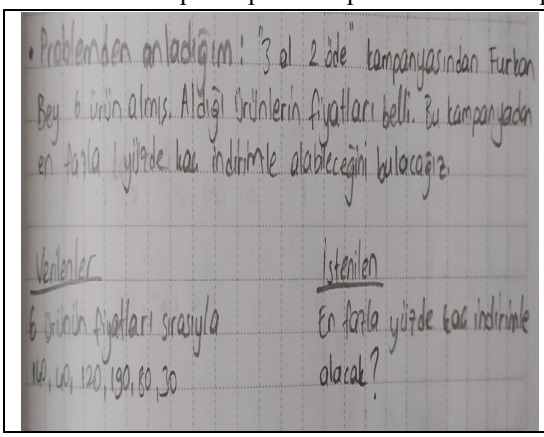
When Table 16 is examined, it is seen that the majority of the participants fully understood the third problem (77%). In addition, it was determined that 8% of the participants understood part of the problem, while the rate of participants who did not understand the problem and did not make any effort was 5%. When the plan preparation category related to the third problem was examined, it was determined that more than half of the participants (74%) developed an appropriate strategy. The rest of the participants either could not determine a correct strategy or did not make any effort to determine any strategy. When the plan implementation category of the third problem was examined, more than half of the participants (66%) reached the appropriate and correct result, while the remaining participants could not reach an appropriate and correct result. When the evaluation category of the third problem was examined, it was found that more than half of the participants (57%) could evaluate, similar to the previous categories, but 36% of the participants had problems in making evaluations. Finally, in the problem posing category, only 44% of the participants were able to pose logical and solvable problems. Almost half of the participants (49%) were found to be unsuccessful in the problem posing process. First of all, the sample participant statement in the problem understanding category of the third problem is given in Table 17.

Table 17: P1 participant's response to the third problem in the category of understanding the problem

 a) problemden anladığım şu şekildi. - En pahalı olan '3' tanesini bir gruba en ucuzlarda bir gruba alalım.	What I understand from the problem is as follows: Let's take the 3 most expensive ones to a group and the cheapest ones to another group.

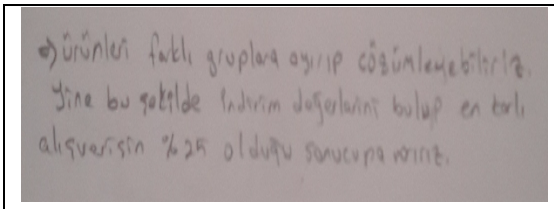
When Table 17 is examined, it is seen that P1 participant does not understand the problem. If what was given and what was requested were clearly written, such a statement would not have been stated. It can be said that the third problem of the P1 participant was unsuccessful in the category of understanding the problem. A successful participant statement for the problem understanding category of the third problem is given in Table 18.

Table 18: P19 participant's response to the third problem in the category of understanding the problem

 • Problemden anladığım: "3 al 2 öde" kampanyasından Furkan Bey 6 ürün almış. Aldığı ürünlerin fiyatları belli. Bu kampanyadan en fazla yüzde kaç indirimle alabileceğini bulacağız. Verilenler: 6 ürün fiyatları sırasıyla 140, 40, 120, 190, 80, 30. İstenilen: En fazla yüzde kaç indirimle alacak?	What I understand from the problem: Furkan bought 6 products from the buy 3 pay 2 campaign. The prices of the products he buys are certain. We will find out how many percent discount he can get from this campaign. Given Prices of 6 products respectively 140, 40, 120, 190, 80, 30 Desired How many percent discount will he get?

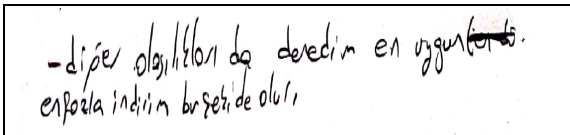
When Table 18 is examined, it is seen that participant P19 understood the third problem correctly. The participant summarized the problem by writing clearly what was given and requested in the problem and successfully passed the plan preparation step. The biggest difficulties encountered in the third problem are in the evaluation and problem posing categories. Participant statements that will set an example for the difficulties experienced in these categories are included.

Table 19: P53 participant's response to the third problem in the evaluation category

	We can divide the products into different groups and analyze them. Again in this way, we find the discount values and conclude that the most profitable purchase is 25%.

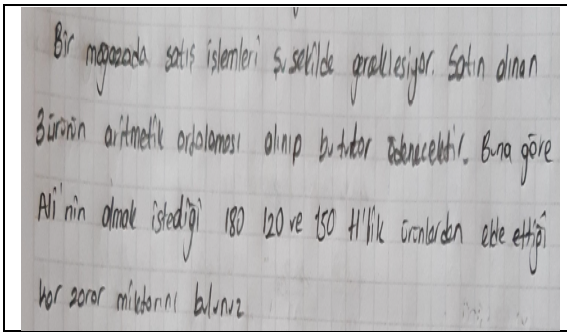
When Table 19 was examined, it was determined that the P53 participant did not make any evaluation. There is no case of reaching any conclusion as stated in the statement. Therefore, the P53 participant was deemed unsuccessful in this category.

Table 20: P1 participant's response to the third problem in the evaluation category

	I tried other possibilities as well, this is the most suitable discount.

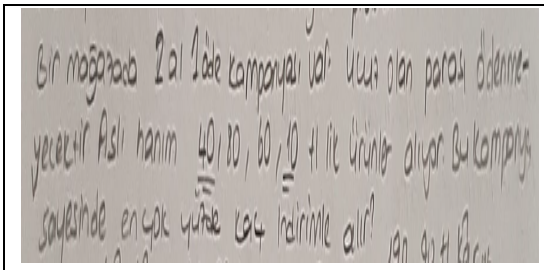
When Table 20 is examined, it is claimed that participant P1 reached the most appropriate result by trying all possibilities. There was no evaluation process and it was only verbally expressed. According to the evaluation criteria, it can be said that the participant failed in this category. Finally, the sample participant response with a logic error in the problem posing category of the third problem is presented in Table 21.

Table 21: P42 participant's response to the first problem in the problem posing category

	Sales transactions in a store take place as follows. The arithmetic average of the 3 purchased products will be taken and this amount will be paid. Accordingly, find the amount of profit and loss that Ali derives from the 180, 120 and 150 TL products he wants to buy.

When Table 21 is examined, it is seen that the P42 participant posed the wrong problem. The reasons for the problem being erroneous are that it is a logic error, there is no solution and no percentage calculation is asked. A successful participant statement for the problem posing category of the third problem is presented in Table 22.

Table 22: P23 participant's response to the first problem in the problem posing category

	There is a buy 2 pay 1 campaign in a store. The cheaper one will not be paid. Aslı buys 40, 80, 60, 10 TL products. Thanks to this campaign, how many percent discount does he get?

When Table 22 is examined, the problem posed by the P23 participant is a logical and solvable problem. Therefore, it can be said that the participant was successful in this category.

4. Discussion

In this study, the answers and explanations of the pre-service teachers to the applied problems were examined and the problem-solving and problem-posing skills of the pre-service teachers were examined. In this context, the aim of the study is to examine the problem-solving processes and problem-posing skills of pre-service teachers, which consists of four stages (understanding the problem, preparing a plan for the solution, applying the plan, evaluating) defined by Polya (1997) with the progressive scoring scale based on the alternative assessment approach. As a result of the analysis, it was determined that the participants showed the highest performance in the category of understanding the problem, and the lowest performance in the category of evaluation and problem posing. Similarly, Deringöl (2006) and Gökkurt, Örnek, Hayat & Soylu (2015) revealed that the students got the highest average score from the problem-solving stage of Polya's problem-solving stages, and the lowest average score from the solution evaluation stage.

When the answers of the participants in the category of understanding the problem were examined in general, it was determined that the problems were generally understood by the participants. When the answers of the participants who had problems in the category of understanding the problem were examined, it was seen that there was difficulty in understanding the problem in cases where what was given and what was requested in the problems were not clearly stated. An important detail here is the category of understanding the problem. If the understanding of the problem is incomplete or erroneous, then a systematic error occurs in the categories of plan development and implementation of this plan. When the answers given in the plan preparation category were examined, it was seen that the participants generally prepared a plan successfully.

When the answers of the participants who have problems in the plan preparation category are examined, it is seen that this question arises from the problem in the category of understanding the problem. Similarly, Karataş and Güven (2004) stated that students made mistakes in the problem solving process due to insufficient understanding of the problem. According to Mayer (1992), the difficulties encountered in the problem solving process stem from the inadequate definition of the problem (understanding the problem and preparing a plan) rather than the mistakes in the solution process. In this context, the results obtained from the study are similar to the results of the mentioned study.

When the answers given by the participants in the plan implementation category are examined, it is seen that the results in this category are in line with the results obtained in the plan preparation category. Participants who were successful in the categories of understanding the problem and preparing a plan were also successful in the category of implementing the plan. However, it was determined that the participants who had problems and made mistakes in the categories of understanding the problem or preparing a plan were also deficient in implementing the plan. Another problem experienced in plan preparation and plan implementation categories is calculation and processing errors. According to the findings, it can be said that the performance of the participants in the evaluation and problem posing process is lower than the other categories. It was determined that participants who failed in the problem posing phase either wrote the same problem or could not write a problem. One of the most important points of being successful in the evaluation category is to understand the problem, prepare a plan to solve the problem and implement this plan correctly. One of the reasons why the success in the evaluation category is lower than the other categories is that the participants do not need to evaluate. No evaluation was made based on the results they found. Karataş and Güven (2004) stated in their study that students who made mistakes in the solution phase of the problem realized their mistakes because they used appropriate strategies and skills in the evaluation phase. In this context, teachers should give students the opportunity to think about the solutions they have made so that students can use the evaluation step effectively in the problem solving process (Gökkurt, Örnek, Hayat & Soylu, 2015).

When the answers given by the participants were examined, it was seen that the problem posing skills of the

participants were not at the desired level. Most of the participants either wrote the problem exactly by changing the numerical values in the given problem or posed illogical problems with no solution. In this context, in order to improve students' problem posing skills, problem posing practices should be made in the lessons, and students should be given feedback on eliminating these mistakes by emphasizing the mistakes made by the students during the problem posing process. Çetinkaya and Soybaş (2018), Dinç (2018), Türnüklü et al. (2017) and Tabak (2019) concluded that students' problem posing skills for operation priority are weak. Ev-Çimen and Yıldız (2018) concluded that students' problem posing skills were at a better level than expected. In this study, it was observed that students had more difficulties in the problem-posing process than in problem-solving. It has been observed that they are significantly more successful in problem solving than in problem posing. In addition, participants showed high performance in other categories than problem posing and evaluation categories. In the study conducted by Bunar (2011), unlike this study, it was concluded that the participants were more successful in the problem-posing process than in the problem-solving process. Gökkurt, Örnek, Hayat and Soylu (2015) concluded in their studies that students were not successful in both problem solving and problem posing.

Another result reached in the study is that the participants had difficulties in expressing the operations in mathematical language. This problem, especially in the process of problem posing, is clearly seen. In the studies conducted by Akarsu-Yakar & Yılmaz (2017) and Yenilmez & Bağdat (2014), it was concluded that the participants had an inability to use the mathematical language, and because of these reasons, they had problems in demonstrating the operations with the mathematical language. In this context, it is similar to the results of the study in question. In this study, it was concluded that the students had language and expression errors while expressing the operation in mathematical language and posing problems for the operation. In the study of Ev-Çimen and Yıldız (2018), similar to this study, it was stated that the problems posed by the students were incorrect in terms of language and expression. Arıkan and Ünal (2013) concluded in their study that most of the students could not pose a problem due to language use.

In this study, problem activities prepared by the researchers were analyzed with a rubric. Different results can be achieved by solving problems with different rubrics. Findings regarding the difficulties or deficiencies faced by pre-service teachers in problem solving and problem posing were analyzed within the framework of Polya's problem solving stages. The same results may not be achieved with another rubric that includes different problem solving stages in the literature. The same is true for the analysis of findings related to problem posing skills. On the other hand, due to the covid 19 pandemic, pre-service teachers received online training on problem solving and setting up, and the advantages and disadvantages of online education and face-to-face education can be revealed by applying the same data collection tools in a process where there is face-to-face education.

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