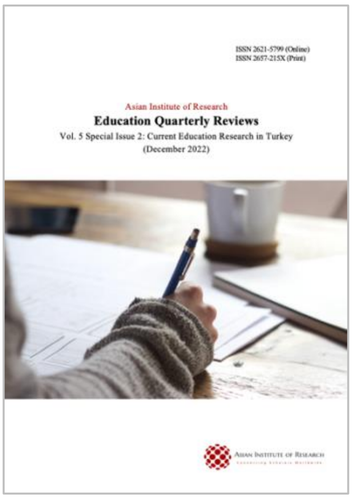

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The Impact of the Cognitive Conflict Approach on the Elimination of the Misconception in Square Root Numbers

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Abstract
This study aims to determine the effect of the cognitive conflict approach on the elimination of misconceptions in square root numbers. For this purpose, this study was conducted with 8th-grade students of a secondary school in a region of Turkey. A three tiered diagnostic test was used to determine the impact of the 5-step cognitive conflict approach in the study. This test was used as a pre-test before applications and the same test was used as a post-test after applications in the same class. The test results of the students were divided into 4 groups called "misconception", "lack of knowledge", "lack of confidence", "scientific knowledge", and these groups were also divided into categories (A, B, ...). 5-step cognitive conflict approach made some transformations between categories. The greatest transformation was the transformation of misconceptions into scientific knowledge. However, it was seen that some misconceptions (B category) continued after the applications. In light of these findings, it is suggested that teachers pay attention to the mathematical language and mathematical representations they use during instruction. Moreover, the topic of root numbers should definitely be explained using the Pythagorean Theorem, and the guidebooks should be revised accordingly.

Keywords: Square Root Numbers, Misconceptions, Three Tiered Test, Cognitive Conflict Approach, Secondary School Mathematics

1. Introduction

Ancient Greek mathematics has reached today's mathematics with revolutionary discoveries and developments. The Pythagorean schools in Croton have shaped history with their work in mathematics. In this school, the theorem was proved that "the square of the hypotenuse length of a right triangle is equal to the sum of the squares of the lengths of the other sides." Until then, those who could express all lengths with rational numbers thought how to express the length of the hypotenuse in a right triangle with right sides 1 unit with the proof of this theorem (Baki, 2014: p.33). In fact, if the length of the sides in a right triangle is 1 unit, the hypotenuse length could not be explained by any of the types of numbers they knew. They called this new type of number that emerged from discussions "irrational numbers," which contradicts reason and logic (Sertöz, 2002; Bingölbali & Özmantar, 2010: p. 12). Today, these numbers, which are also referred to as root numbers, for example, $\sqrt{2}$ the number 1.41421... have infinite digits and $\sqrt{x^2}$ is defined as = IxI. In the MEB (Ministry of Education) textbook (Böge & Akıllı, 2019), the square root calculation begins by finding the side lengths of squares and comparing them to their areas. Then the square root operation continues with the process of figuring out which number is the square of a non-negative number. Irrational numbers, which mathematicians struggled with in ancient times, are topics students still have difficulty understanding today. Bagni (2000) surveyed high school students between 16 and 19 and attempted to identify their errors in his study. As a result of this study, it has been observed that students have errors such as $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$ and $\sqrt{x^2}$ = x. In a study by Crison (2012), when asked about the value of $\sqrt{32}$, students gave the answer $4\sqrt{2}$. When asked about the value of $\sqrt{16}$, they answered ±4. Roach, Gibson, and Weber (2004) asked first-year university students about the value of $\sqrt{25}$, most of the students answered $\sqrt{25}$ = ± 5. They attributed the reason for students' misconceptions on this subject to the frequent expression; if $x^2 = 25$, $x = \pm 5$ in algebra textbooks. Negative root numbers ($-\sqrt{25}$) become complete chaos for students who think this way. It turns out that there are other misconceptions about radical expressions among both students and teacher candidates. According to Sirotic (1998), some prospective mathematics teachers have difficulty understanding which number $\sqrt{5}$ can be equal to or estimating between which numbers it lies (cited in Duatepe-Paksu, 2008). In this case, Sirotic and Zazkis (2007) suggest seeing the value of $\sqrt{5}$ on the number, line using the Pythagorean theorem.

As can be seen, the ongoing errors in root numbers continue to this day, and the search for pedagogical solutions continues. There are several paths that the teacher should take. One of these ways, the didactic approach (Swan, 2001), is that the error made by the student is communicated directly to the student, and the teacher corrects the error. Thus, if there is an error, an attempt is made to correct it. Another approach is cognitive conflict (CC) (Simon, Tzur, Heinz & Kinzel, 2004). In this approach, the student notices his error in the questions asked by the teacher. The student, noticing the discrepancy and contradiction between his previous answer and his new answer, finds the answer through reflection and reasoning and moves away from his error with his internal calculation. The student's error or misconception is discussed in class and turned into an advantage (Zembat, 2008). The only way to prevent a permanent misconception is to discuss and communicate the misconception and to each other. Although the issue of dealing with misconceptions in class is controversial, this problem should be discussed and resolved in class without exposing or offending students (Bingölbali & Özmantar, 2010, p.23). By discussing and eliminating misconceptions in the classroom, the existing misconceptions can be corrected, and possible misconceptions can be prevented. If the teacher starts a new topic before revealing and eliminating the misconceptions, the student will not be aware of the mistake; maybe student will transfer student's mistake to the new topic and even make new mistakes. Therefore, when errors are detected in students during teaching, they should be intervened immediately, and these errors should be eliminated as soon as they are detected. For this reason, this study, which aims to eliminate some critical misconceptions about root numbers, is considered an original and important study in terms of the approach and activities it offers.

The study investigated what kind of situation arises when a teacher who tries to eliminate the misconceptions about square root numbers in class using a didactic approach instead tries to eliminate the misconceptions using the CC approach. For this, cognitive conflict approach was used and activities suitable for this approach were designed. The research question is: What is the effect of the CC approach in overcoming misconceptions related to square root numbers?

1.1 Theoretical Framework

In the "CC" theory presented by Festinger in 1952, he mentions that man tends to be always consistent. According to Festinger's CC theory, "If one belief, knowledge, or attitude requires the opposite of another belief, knowledge, or attitude that a person holds, there is a CC between those two beliefs, knowledge, or attitude. According to this theory, since such CC's are stressful situations, the person will be motivated to eliminate this conflict" (Kılınç & Torun, 2011: p.4). For example, if a person acknowledges another person as very good and later witnesses an evil committed by this person he knew as good, he begins to experience CC.

CC is based on the theory of the constructive approach. One of the pioneers of cognitive structuralism, Piaget, explains learning in adaptation, assimilation, and balancing processes. If the new information does not conflict with the individual's prior knowledge, it is accepted, and a new cognitive balance is formed. If the new information contradicts prior knowledge, confusion, an imbalance occurs because the new information cannot be integrated into the existing structure. The individual strives to eliminate this imbalance. By adapting to this situation, he understands that he is confronted with new learning and creates a new cognitive structure (Gelebek, 2011: p.56).

Hewson and Hewson (1984) defined CC as one of the classical models of conceptual change strategies. CC is an important stage of conceptual change proposed by Posner et al. (Posner, Strike, Hewson & Gertzog, 1982). Posner et al. (1982) argue that learning begins with a need and occurs when people are unsatisfied with their existing thoughts. The student experiences dissatisfaction when the student finds that the information student believes to be true is inconsistent with scientific knowledge. This disturbing situation in the student turns into a conflict. The desire to get rid of this situation prompts the student to research and learn. However, the students' willingness to deal with CC and find a solution is not uniform. It is difficult for all students to reach a significant CC (Chan, Burtis & Bereiter, 1997). This is because each student's alternative conceptions are different, and student self-management and diversity limit the transition from one body of knowledge to another, i.e., conceptual change. To overcome misconceptions, conceptual change strategies should make students understand that their concepts are wrong. For this reason, creating CC is an essential prerequisite for realizing conceptual change (Lee, Kwon, Park, Kim, Kwon, & Park, 2003). Posner et al. (1982) listed the four steps of the teaching method CC: 1) creating a conflict, 2) elaborating the conflict, 3) presenting the new idea, and 4) showing that this new idea works. Limon (2001) listed the steps as 1) conflict building with abnormal data, 2) using analogies to force students to change, and 3) collaborative learning to promote discussion. Lee et al. (2003) explain the steps of CC as follows: 1) the introduction is made through the presentation of a CC, 2) the conflict is revealed through demonstration activities that involve an assimilation and accommodation process, and 3) the resolution is concluded with a discussion. According to Baser (2006), learning with CC is done as follows: 1) activating alternative concepts in students, 2) presenting situations that cannot be explained by existing concepts, 3) creating CC with abnormal situations, 4) introducing new concepts that are needed, 5) making students actively build knowledge, 6) students interact with each other to share their ideas about abnormal situations, and 7) trying to see if these ideas help solve the problem that may occur in the same way in the future. Mufit, Festiyed, Fauzan, and Lufri (2018) designed the prototype of CC-based learning. This design includes 1. Activation of conflicts (prejudice), 2. Presentation of CC, 3. Discovery of concepts, and 4. Reflection. Yaman (2013) applied the CC approach in 6 steps. 1. Activation of Misconception, 2. Presentation and Acquisition, 3. Deepening the Conflict, 4. Presentation of Theoretical Information, 5. Extension / Expansion of the Concept, 6. Conclusion – Roundup. Okumuş (2022) used the CC approach in 5 stages. These:

1.Misconceptions are revealed and exhibited

2.Cognitive conflict is created. By giving contradictory examples of each mistake, the student is provided to experience conflict.

3.The reasons for the mistake are mentioned and scientific information is explained. In doing so, the concept change text, the rebuttal text, etc. techniques can be used.

4.Whether the error persists is tested with other examples. If the error continues, it is returned to the second stage and a contradiction is created.

5.A conclusion is drawn and an evaluation is made.

1.2 Studies carried out

Creating CC is difficult and may not produce the same result for every student. Zohar and Aharon Kravetsky (2005) investigated whether the effect of CC on conceptual achievement changes when students at different academic levels use different learning strategies. The result of the study was that students with high academic achievement benefited from the CC teaching method, while the direct teaching method hindered their progress. In contrast, students with low academic achievement benefited from the direct teaching method, while the CC method hindered their progress. In other words, the CC method produced different results for students with various academic performances. There are some difficulties and limitations related to CC (Chan, Burtis, & Bereiter, 1997; Demastes, Settlage, & Good, 1995; Dreyfus, Jungwirth & Eliovitch, 1990; Limon, 2001; Zohar & Aharon-Kravetsky, 2005). Many of the difficulties associated with implementing the CC strategy are closely related to the complexity of the factors that affect conceptual learning in the classroom context. First, the success of the CC approach depends mainly on the individual student to identify and resolve the conflict (Limon, 2001). The issue here is that intellectually less gifted students may not recognize the conflict and that some of those who do recognize it may not be able to overcome it even if they do (Planinic, Krsnik, Pecina & Susac, 2005). Second, students are often reluctant to give up the alternative ideas resulting from their own experiences (Chan et al., 1997; Planinic et al., 2005). In addition, scientific concepts are often complex for students to understand because they are not based on their experience (Planinic et al., 2005). Therefore, students who cannot absorb scientific concepts with their experiences may learn inadequately and lack confidence. There are controversial results regarding the effectiveness of CC in learning scientific concepts. According to Zohar & Aharon-Kravetsky (2005), students have superficial disagreements than the more radical changes implied by conceptual change theory. Students' concepts are often distinguished as alternative or scientific concepts in CC. According to this view, some students do not understand the inconsistent phenomena in complex concepts and maintain their alternative concepts (Chabay & Sherwood, 2006). Therefore, in the classical model of conceptual change, the ability of CC to bridge the gap between students' alternative understandings and scientific concepts is limited. In general, CC of the classical conceptual change model has been proposed and presented as a linear, rational, deterministic step and revolutionary (Nussbaum & Novice, 1982).

The CC approach is based on cognitive ideas and seems to be more effective in learning its concepts than the traditional transfer approach (Duit & Treagust, 2003; Posner et al., 1982). Some studies have shown that conceptual change in physics cannot be achieved with traditional instruction alone (Dykstra, Boyle & Monach, 1992; Grayson, 1994; Hake, 1998; Dega, Kriek & Mogese, 2013).

Shahbari and Peled (2015) conducted a study examining the conceptual changes of students using the French fries model to break with the CC strategy. They studied with experimental and control groups. The experimental groups were successful in a realistic modeling situation in which they had first to increase and then decrease the size of a bag of French fries. As a result of this study, it was found that there was a conceptual change, and the changing reference was better understood. Liang (2016) used the CC strategy when teaching the limit topic with graphing calculators. This study enabled students to identify their misconceptions using the graphing calculator to create a CC in limited situations as x approached infinity. As a result of this study, he found that the graphing calculator plays an essential role in forming CC. However, he also pointed out some limitations. First, the CC strategy was not sufficient for conceptual change. No matter how convincing the new evidence appeared, it was difficult to abandon the old thinking. Second, the CC strategy enabled the recognition of misconceptions rather than the construction of new ideas (as with other methods, e.g., multiple representations, analogies). Third, CC is not universally applicable in all teaching situations. In some cases, it works; in others, it does not. For example, graphical representations work better when you are teaching quadratic functions. Fourth, the CC strategy requires more time than traditional methods. Because teachers have limited instructional time, extending instruction for a longer period is difficult. Irawati et al. (2018) used the ADDIE (Analysis, Design, Development, Implementation, and Evaluation) model in their study of reducing student misconceptions in algebra with a CC strategy. They applied this model to students, which they had developed using the CC strategy. As a result, they succeeded in minimizing students' errors in addition and subtraction in algebraic expressions using the CC strategy. However, it was found that some students still had misconceptions. Okumuş (2022) found in her study that the 5-stage cognitive conflict approach reduced the misconceptions about percentages. But one student resisted to change the misconception and continued the misconception.

It is a difficult task to eliminate misconception. This has been expressed in studies conducted with CC. CC leads to different results for each student. Nevertheless, learning the concepts of CC based on cognitive ideas seems to be more effective than the traditional transfer approach. Students will find it easier to give up their alternative concepts if they are given confidence and willingness, if they have the opportunity to experience their ideas, and if they see that their new concepts work. In this context, our study aims to investigate the impact of the 5SCC approach in eliminating misconceptions related to square root numbers. There is no previous study on this topic. We think that our study will fill this gap and contribute to teachers and students in eliminating misconceptions.

2. Method

The research is a particular case study that examines the effect of the 5SCC approach on overcoming student's misconception abouth the square root number. A specific case study is an empirical research method that deals with a current phenomenon within its real-world setting, where the boundaries between the phenomenon and the content at stake are not clear, and is used when there is more than one source of evidence or data (Yin, 1984: 23, as cited in Yıldırım & Şimşek, 2000). Yıldırım and Şimşek (2000) stated that the unique case study is a method based on "how" and "why" questions that enables the researcher to investigate in-depth a phenomenon or event that the researcher cannot control. In this sense, the case study is an appropriate method for our study of how the 5SCC approach affects misconceptions.

2.1 Study Group

In a secondary school in a region of Turkey, practices were conducted in a class of 20 eighth-grade students and with the mathematics teacher of that class.

2.2 Processing

The necessary permissions for this study were obtained by the Recep Tayyip Erdoğan University Publication Ethics Committee with the decision numbered 2021/239 on 16.11.2021. Root numbers are the first topic of unit 2 of 8th grade. First of all, in 2nd semester 8th grade, radical numbers were explained to the students with the traditional method in a classroom chosen by the teacher at random. Then the Pythagorean theorem was explained. Afterwards, the diagnostic test was applied as a pre-test for this study. Then rooted numbers were explained with the 5SCC approach (Okumuş, 2022) by teacher. Immediately after this application, which lasted 4 lesson hours, the diagnostic test was administered to the students as a post-test.

2.2.1. Data Collection Tool

This study used a three tiered diagnostic test for square root numbers developed by Baki, Güveli & Güveli (2019). The diagnostic test, which includes the following gains, consists of 3 steps. In the first step, the student is asked to choose one of the options, in the second step to explain why they chose that option, and in the third step to indicate whether they are sure of that answer. It is believed that three tiered tests are better at detecting students' misconceptions than one- and two-tiered tests (Baki et al., 2019). Therefore, it was considered appropriate to use this three tiered test to detect misconceptions about square root numbers in this study. For the tier 1, the average item difficulty is 0.66, discrimination is 0.32, and Cronbach's Alpha reliability coefficient is 0.59. For the tier 2, the average item difficulty was 0.58, the discrimination was 0.40, and the reliability coefficient Cronbach alpha was 0.72. For the tier 3, the average item difficulty was 0.56, the discrimination was 0.42, and the reliability coefficient of Cronbach alpha was 0.77.

2.2.2. CC stages

The mathematics course was taught according to the following stages of the 5SCC approach:
1) Revealing misconception: In the first step, the misconceptions revealed by the concept diagnostic tests are written on the board. It is not said to which student they belong. E.G., The number closest integer to $\sqrt{23}$ is 22 (The misconception that the square root makes the number smaller).
2. Creation of a Conflict: The teacher creates a cognitive conflict in this step. E.G., If the square root expression makes the number smaller, then $\sqrt{1} < 1$ or $\sqrt{\frac{1}{4}} < \frac{1}{4}$. But we know that this is not so. There is a contradiction here.
3) Reasons for the error are given, and scientific information is explained. E.G., It is explained between which two integers $\sqrt{23}$ lies.
4) More examples are used to test whether the error persists or not. If the error persists, the teacher returns to the second step, generating a conflict. E.G., It is asked what two integers $\sqrt{23} + \sqrt{2}$ are in between. If the false assumption persists;
E.G.: $\sqrt{23} + \sqrt{2} = \sqrt{25}$. If it is true, then $\sqrt{16} + \sqrt{9} = \sqrt{25}$. But we know that $\sqrt{16} = 4$, $\sqrt{9} = 3$ and $\sqrt{25} = 5$ (7=4+3). There is a conflict here.
5) The conclusion is drawn, and the evaluation is made.
Concerning the course, at no time are negative comments made about what students say or think. This approach aims to enable students to find the correct concept by pointing out the contradictions in their alternative concepts.

2.4. Data Analysis

The test on the square root number were evaluated and analyzed according to the categories indicated in the following table.

Table 1: Diagnostic test evaluation categories

Tier 1	Tier 2	Tier 3	Categories
True	True	I am sure	A
True	False	I am sure	B
False	False	I am sure	C
True	False	Not sure	D
False	False	Not sure	E
False	True	Not sure	F
True	True	Not sure	G
False	True	I am sure	H

In the literature, the A category is called "scientific knowledge", the B category is called "false positives", "misconception" or "true respond with false reason", the C category is called "misconception", the D, E, F categories are called "lack of knowledge", the G category is called "lack of confidence", "lack of knowledge" or "lucky guess", the H category is called "false negatives", "misconception", "false respond with true reason" or "lack of knowledge" (Kırbulut & Geban, 2014; Çiğdemoğlu & Arslan, 2017; Bozdağ, 2017; Bulut, Güveli & Güveli, 2021). In this study, the A category was called "scientific knowledge", the B and C categories were called "misconception", the D and E categories were called "lack of knowledge", the G category was called "lack of confidence". Since there were no students answer in the F and H categories, these categories were not taken into consideration.
The pre-post test results were compared with the percentages and frequencies of the assessment categories. Students were coded as S1, S2,...

3. Results

Three tiered test administered to students before and after the activities using the 5SCC approach. According to the test results, all categories were divided into 4 groups as scientific knowledge, misconception, lack of confidence, lack of knowledge and lack of confidence. The percentages and frequencies of these groups according to the pre-post test results are given in Table 2.

Table 2: Pre-post test results according to the groups

Items	Scientific knowledge(A)		Misconception (B, C)		Lack of knowledge(D,E)		Lack of confidence G	
	Pretest	posttest	Pretest	posttest	Pretest	posttest	Pretest	posttest
1	18 (90%)	19 (95%)	1 (5%)	-	1 (5%)	-	-	1 (5%)
2	18 (90%)	19 (95%)	1 (5%)	-	1 (5%)	1 (5%)	-	-
3	19 (95%)	19 (95%)	1 (5%)	-	-	-	-	1 (5%)
4	19 (95%)	19 (95%)	-	-	1 (5%)	1 (5%)	-	-
5	19	19 (95%)	-	-	1 (5%)	-	-	1 (5%)

	(95%)							
6	14 (70%)	17 (85%)	2 (10%)	-	-	3 (15%)	4 (20%)	-
7	16 (80%)	17 (85%)	3 (15%)	3 (15%)	1 (5%)	-	-	-
8	2 (10%)	16 (80%)	13 (65%)	2 (10%)	5 (25%)	2 (10%)	-	-
9	2 (10%)	16 (80%)	15 (75%)	3 (15%)	3 (15%)	-	-	1 (5%)
10	-	19 (95%)	16 (80%)	1 (5%)	4 (20%)	-	-	-

According to the table 2, there is an increase in favor of the post-test in the scientific knowledge group. There is a decrease in favor of the post-test in the misconception group. There is a decrease in favor of the post-test in the lack of knowledge group. The lack of confidence, which was seen only in the 6th question in the pre-test, was not seen in the post-test. However, in some questions (1th, 3th, 5th, 9th) that were not encountered in the pretest appeared. On the other hand, the percentile between the pre-test and post-test did not change in the lack of confidence.

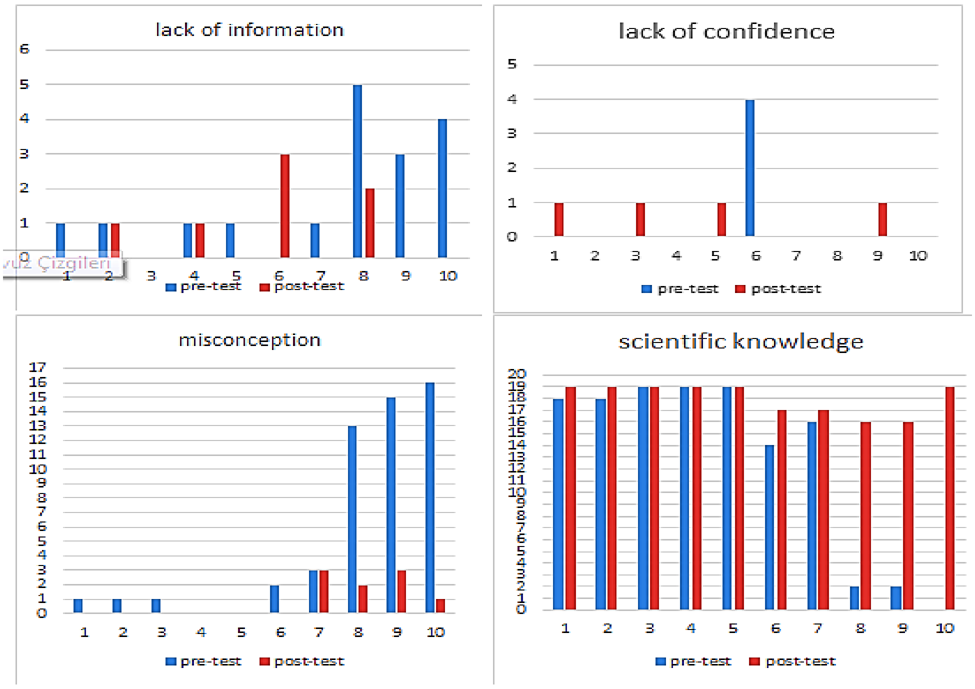


Figure 1. Graphs of pre-posttest results

Table 3 shows the change the groups according to the categories within the group.

Table 3: Pre-posttest results according to the B, C, D, E categories

	Misconception				Lack of knowledge			
	B		C		D		E	
	pre	post	pre	post	pre	post	pre	post
İtem 1	5%	-	-	-	-	-	5%	-
	(1)	-	-	-	-	-	(1)	-

İtem 2	5% (1)	- -	- -	- -	- -	- -	5% (1)	5% (1)
İtem 3	- -	- -	5% (1)	- -	- -	- -	- -	- -
İtem 4	- -	- -	- -	- -	- -	- -	5% (1)	5% (1)
İtem 5	- -	- -	- -	- -	- -	- -	5% (1)	- -
İtem 6	5% (1)	- -	5% (1)	- -	- -	15% (3)	- -	- -
İtem 7	- -	15% (3)	15% (3)	- -	- -	- -	5% (1)	- -
İtem 8	- -	10% (2)	65% (13)	- -	- -	5% (1)	25% (5)	5% (1)
İtem 9	10% (2)	15% (3)	65% (13)	- -	- -	- -	15% (3)	- -
İtem 10	5% (1)	5% (1)	75% (15)	- -	5% (1)	- -	15% (3)	- -

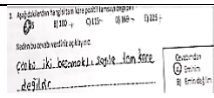
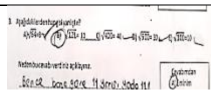
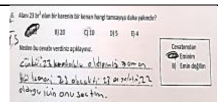
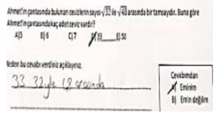
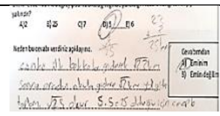
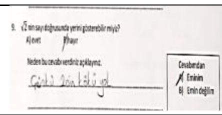
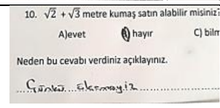
According to the table 3, there is an increase in favor of the post-test in the category B of misconception. This shows that there is an increase in students' true answers with false reasons that they are sure of. In the post-test, there were no students in the category C of misconception. This shows that there are no false answers given with false reasons that students are sure of. There is an increase in favor of the post-test in the category D of lack of knowledge. This shows that there are an increase in students' true answers with false reasons that they are not sure of. There is a decrease in favor of the post-test in the category E of lack of knowledge. This shows that there are a decrease in students' false answers with false reasons that they are not sure of. In the posttest, some transformations caused the increase and decrease between the categories (Appendix A). These transformations frequencies are given in Table 4.

Table 4: The transformation frequencies of the categories

transformation		frequencies	explanation
Pretest	posttest		
C	A	32	positive transformation
E	A	11	positive transformation
C	B	9	positive-inadequate transformation
B	A	5	positive transformation
G	A	3	positive transformation
E	G	2	positive-inadequate transformation
C	G	2	positive-inadequate transformation
C	D	2	positive-inadequate transformation
B	D	1	positive-inadequate transformation
G	D	1	negative transformation
D	A	1	positive transformation

As can be seen from the table 4, category B was transformed into categories A and D. G category, was transformed into categories A and D. Category C was transformed into categories A, B, G and D. Category D was transformed into category A. Category E was transformed into categories A and G. The biggest transformation was from category C to category A. The misconceptions revealed in the pre-test are given in the table below.

Table 5: Misconceptions in the pre-post test

Misconceptions	Test items		Examples of students' papers
	Pretest	Posttest	
1.Determine the root number by the number of digits	1	-	
2.The root number is divisible by 2	2,3	-	
3.The root expression makes the value of a number smaller	6	-	
4.Ignoring the square symbol	7,8,10	-	
5.Adding two separate root numbers under the same root	8,10	8	
6.The numbers that do not go out of the root do not have an exact place in the number line	9	9	
7.Root number cannot be measured	10	10	 10. $\sqrt{2}+\sqrt{3}$ metre kumaş satın alabilir misiniz? A)evet B)hayır C)bilm Neden bu cevabı verdiniz açıklayınız.

The students continued the following misconceptions after the practices. These misconceptions:

- Adding two separate root numbers under the same root.
- The numbers that do not go out of the root do not have an exact place in the number line.
- Root number cannot be measured.

4. Conclusion and discussion

4.1 Change in scientific knowledge

The pre-post test results show that scientific knowledge remained the same for some questions, while it increased sharply for others. The reason for this increase is the conversion from misconception (C, B categories), lack of knowledge (E, D categories) and lack of confidence (category G) to scientific knowledge (A). This transformation shows that 5SCC effectively increased the scientific knowledge. Okumuş (2022) found that after the cognitive conflict approach applied to the students, the scientific knowledge levels of the students increased in the posttest. This result is in agreement with our study.

Sirotic and Zazkis (2007), using the Pythagorean theorem, suggest seeing the value of root numbers on the number line. This is the best way to learn root numbers. In MEB textbook (Böge & Akıllı, 2019), we start from area calculations and begin with the example "If the area of a square is 4, then one side is 2." and root numbers are thought of using examples such as $\sqrt{4}$ = 2, $\sqrt{9}$ = 3 etc.. This gets students thinking that the root is an operation that converts a number to an integer. If it cannot be converted, this leads to a misunderstanding such as "There is no such number, it has no place in daily life, cannot be used, cannot be measured." However, if activities are performed using the Pythagorean Theorem to show that they have a place on the number line, it can be declared that such numbers exist, have a place in daily life, and can be used. We think that 5SCC is effective because the activities are done using the Pythagorean Theorem.

4.2 Change in misconceptions

Misconceptions have completely disappeared in category C according to the posttest results. Misconceptions in category C turned into B, G, D and mostly A. However, category B showed a slight increase in favor of the posttest. The reason for this increase is positive-inadequate transformation from category C to category B. In other words, the students turned their false answers with false reasons, which they were sure of, into true answers with false reasons. This shows that although the students gave the true answer to the first tier, they still do not know the reason. This shows that 5SCC activities are effective in eliminating misconceptions in category C, but not in category B.

The following three misconceptions remained at the end of the applications.
1) Adding two separate root numbers under the same root
2) The numbers that do not go out of the root do not precisely place in the number line.
3) Root number cannot be measured

The misconception that two separate root numbers are added under the same root is a common misconception found in some studies (Bagni, 2000; Özkan, 2011, İşleyen & Mercan, 2013; Baki et al., 2019). Many students think that root numbers do not precisely place on the number line because root numbers do not give an exact value (Baki et al., 2019). Perhaps students think root numbers are immeasurable because they do not use them in currencies and weight calculations. Students have difficulty attaching meaning to situations they do not see daily. In this case, students either develop alternative concepts or are unfamiliar with the situation. However, students should know that; if √2 indicates a length and $\sqrt{3}$ indicates a length, then $\sqrt{2}$ + $\sqrt{3}$ also indicates a length and is displayed on the number line. An example of a square fabric with a side of 1 meter can be given from daily life. We cut the fabric on its diagonal. We can continue with this length and get √5 meters of fabric.

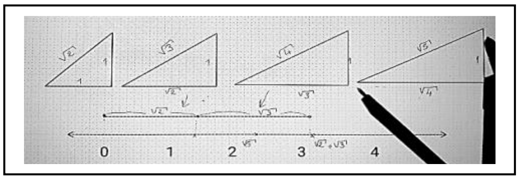


Figure 2: Finding the length of $\sqrt{2}$ + $\sqrt{3}$

Irawati (2018) found that although the CC method minimizes student misconceptions, misconceptions persist for some students. Okumuş (2022) found that the CC approach greatly reduced students' misconceptions, but a student still maintained her misconception. Our study comes to a similar conclusion. Acceptance and internalization is a time-consuming and challenging process. Even if the new knowledge is convincing, it is challenging for students to abandon their old concepts (Liang, 2016). It takes more time for the misconceptions to be eradicated. In the meantime, students should be constantly exposed to and repeat new concepts. Teachers should ensure that students do not have misconceptions and receive good training from the beginning. Cangelosi et al. (2013) point out the importance of language and the notation(s) used in developing conceptual understanding in students. Teachers should pay attention to their expressions and mathematical representations depending on their language in the classroom. If care is not taken, students will either not understand or ignore the root symbol ($\sqrt{\quad}$) in the root number. In this way, the student comes to the brink of misconception due to limited perception and gives up on more challenging topics they will encounter in the future. If there is a misconception, he should look for opportunities to take action and eliminate it. The 5SCC activities, supported by examples from daily life, have not eliminated misconceptions, but they have greatly reduced them.

4.3 Change in the lack of knowledge

The lack of knowledge (category E) was the same in some questions; while it was low in some questions, it (category D) increased in some questions. One of the reasons for this increase is the transformation from misconceptions (C, B categories) to lack of knowledge (category D). Lack of knowledge is a better situation than misconception. Because, in the lack of knowledge, student's current knowledge is not sufficient to solve the problem. This shows that the student is open to change and can be persuaded. However, the student with a misconception is sure of the concept of alternative and it is difficult to give up on the concept that the student is sure of. This transfer is positive but not adequate. Because, it could not completely eliminate the lack of knowledge. For this, it is recommended to make 5SCC events more interesting. It can become even more interesting if concept cartoons, computer animations, etc., are added to the 5SCC events. In this way, students can acquire knowledge eagerly. The second reason for the increase is the transformation from a lack of trust (category G) to a lack of knowledge (category D). The true answers that the student was unsure of turned into a true answer with an false reason that the student was unsure of. The student's lack of confidence in knowledge caused the student to easily change the knowledge and make mistakes. The reason for the negative transformation was may be that the student answered by rote in the pre-test and made a lucky guess in the post-test. The student did not participate in the activities, did not listen well, was not ready to learn. In the study of Okumuş (2022), in which a similar result was seen, this situation is explained by the fact that the student is go-come living with the CC approach and is not sure of her answer.

4.4 Change in lack of confidence

While the lack of confidence disappeared completely in one question, it appeared in some questions. Misconceptions (category C) and lack of knowledge (category E) in some questions turned into a lack of confidence (category G) in the post-test. The students succeeded in converting the false answers that they were sure or unsure of into true answers. In the study of Okumuş (2022), the misconception and lack of knowledge in the pre-test turned into a lack of confidence in the post-test. This was positive but inadequate. Because, they could not be sure of the answers. Confidence in their knowledge has not been fully formed and they have not fully assimilated the new knowledge. Perhaps, they were perplexed by the conflict they experienced and skepticism. When students change their alternative concepts, they cannot internalize the new concept immediately. For students who change their alternative concepts, it is challenging to internalize new concepts and requires a long process. Therefore, teachers should be stable and encourage the student who replaces alternative concept with the new concept to do more exercises and practice. It will be easier for the student to gain experience through practice and adopt the new concept as they gain experience. The teacher must also encourage and reward the student with words of praise.

It is thought that it can be more effective if concept change texts or concept cartoons are added to the activities carried out with the 5SCC approach. If more activity-rich environments are created and students are given some time, misconceptions and lack of knowledge will be reduced even more.

This study will provide an idea and give researchers a guide for eliminating misconceptions about root numbers and teaching the topic using the approach CC. It will present teachers with a different approach to eliminating students' misconceptions internationally.

The study is limited to the acquisition of which two natural numbers a non-square square root number is between. Future studies may investigate the effect of 5SCC in eliminating misconceptions in various mathematical subjects.

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Appendix A

Pretest and post test results according to the categories

Table A1 shows each student's categories in each item of the test and the transformation of these categories in more depth.

Table A1: Pre-post test results according to the categories

	İtem 1		İtem 2		İtem 3		İtem 4		İtem 5		İtem 6		İtem 7		İtem 8		İtem 9		İtem 10	
	pre	post	pre	post	pre	post	pre	post	pre	post	pre	post	pre	post	pre	post	pre	post	pre	post
S1	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	E	A	C	A
S2	A	A	A	A	A	A	A	A	A	A	A	A	A	A	E	A	C	A	C	A
S3	A	A	A	A	A	A	A	A	A	A	G	A	A	A	E	A	E	A	C	A
S4	A	A	A	A	A	A	A	A	A	A	G	A	A	A	A	A	B	A	E	A
S5	B	A	A	A	A	A	A	A	A	A	B	D	A	A	C	A	A	A	C	A
S6	A	A	A	A	A	A	A	A	A	A	A	A	A	A	C	A	C	B	C	A
S7	A	A	B	A	A	A	A	A	A	A	A	A	A	A	C	A	C	A	C	A
S8	E	G	A	A	A	A	A	A	A	A	G	A	E	A	C	A	A	A	D	A
S9	A	A	A	A	A	A	A	A	A	A	A	A	A	A	C	A	C	B	C	A
S10	A	A	A	A	A	A	A	A	A	A	A	A	C	B	C	A	E	A	C	A
S11	A	A	E	E	A	A	A	A	E	G	A	A	A	A	C	A	C	A	C	A
S12	A	A	A	A	A	A	A	A	A	A	A	A	C	B	C	A	C	A	C	A
S13	A	A	A	A	A	A	A	A	A	A	A	A	C	B	C	A	C	A	C	A
S14	A	A	A	A	A	A	A	A	A	A	A	A	A	A	C	B	C	A	C	B
S15	A	A	A	A	A	A	A	A	A	A	A	A	A	A	C	A	B	A	B	A
S16	A	A	A	A	A	A	E	E	A	A	A	A	A	A	E	A	C	A	E	A
S17	A	A	A	A	A	A	A	A	A	A	A	A	A	A	C	D	C	A	C	A
S18	A	A	A	A	C	G	A	A	A	A	C	D	A	A	E	E	C	G	E	A
S19	A	A	A	A	A	A	A	A	A	A	G	D	A	A	E	A	C	B	C	A
S20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	C	B	C	A	C	A