

Education Quarterly Reviews

Akbaş, Elif Ertem. (2021), Examining the Structure of Observed Learning Outcomes of Associate-Degree Vocational School Students in a CAS-Supported Environment: Limit-Continuous Sample. In: *Education Quarterly Reviews*, Vol.4, No.2, 312-328.

ISSN 2621-5799

DOI: 10.31014/aior.1993.04.02.282

The online version of this article can be found at:
<https://www.asianinstituteofresearch.org/>

Published by:
The Asian Institute of Research

The *Education Quarterly Reviews* is an Open Access publication. It may be read, copied, and distributed free of charge according to the conditions of the Creative Commons Attribution 4.0 International license.

The Asian Institute of Research *Education Quarterly Reviews* is a peer-reviewed International Journal. The journal covers scholarly articles in the fields of education, linguistics, literature, educational theory, research, and methodologies, curriculum, elementary and secondary education, higher education, foreign language education, teaching and learning, teacher education, education of special groups, and other fields of study related to education. As the journal is Open Access, it ensures high visibility and the increase of citations for all research articles published. The *Education Quarterly Reviews* aims to facilitate scholarly work on recent theoretical and practical aspects of education.



ASIAN INSTITUTE OF RESEARCH
Connecting Scholars Worldwide

Examining the Structure of Observed Learning Outcomes of Associate-Degree Vocational School Students in a CAS-Supported Environment: Limit-Continuous Sample

Elif Ertem Akbaş¹

¹ Van Yuzuncu Yıl University Faculty of Education

Abstract

This study aimed to learn the learning outcomes of associate-degree students attending a Vocational School (VS) in a CAS-supported learning environment within the scope of the limit-continuity subject. The study was conducted using the action research method, and the worksheets prepared by Ertem Akbaş (2016) were used. While evaluating and interpreting the VS students' learning outcomes, the SOLO taxonomy was preferred. The study group included 32 VS associate-degree students in Turkey. Within the framework of the research problem, detailed information was provided about what level of the SOLO taxonomy the students' learning outcomes corresponded to. The learning outcomes of the VS students were found to be below the relational structure level according to SOLO taxonomy in the environment where the CAS software was used. Thanks to the CAS software, the quality of the pre-structure level and uni-structure level learning outcomes of VS students increased to and over the multi-structure level.

Keywords: Vocational School Associate-Degree Students, CAS, SOLO, Limit-Continuity

1. Introduction

With the development of technology in the field of education, it is not difficult to predict that the future teaching method will be shaped and improved in the light of technological developments. This has revealed new expectations in the teaching methods used in mathematics lessons. According to these expectations, students will be able to discover technology and mathematics on their own, and paper-pencil applications will remain in the background (Papert, 1993). Even if the expectations did not come true, this universal dimension provided by technology had an important effect on 'what and how should we teach?' within the content of mathematics. Parallel to this problem, important experience has been gained on how to use a computer so that students can learn mathematics better (Powers & Blubaugh, 2005; Pierce & Stacey, 2002; Renshaw & Taylor, 2000; Sevimli & Delice, 2015; Wiest, 2001). Many researchers stated that Dynamic Geometry Software (DGS) and Computer Algebra Systems (CAS) have a significant potential in reflecting these experiences into mathematics classes (Hazzan & Goldenberg, 1997; Powers & Blubaugh, 2005; Tabuk, 2019). Hazzan and Goldenberg (1997) pointed

out that geometry software, which has a dynamic feature, provides students with the opportunity to concentrate much more on abstract structures than the commonly used paper and pencil works. Students' concentration on abstract structures leads to an increase in the imagination power in mathematics and means opening the way of intuition and thus the ways of learning. These ways will improve students' skills such as doing analysis, making assumptions, making generalizations and problem solving (Baki, 1994). In this respect, it is important to making students active producers of information and rather than making passive receivers of information by considering student-centeredness in teaching.

One of the few sub-branches that constitute the basis of contemporary mathematics is analysis. The definitions of topics such as derivative, integral and approximation theory, which are all included in the scope of the course of analysis, are built on the formal definition of the concept of limit and continuity. The concept of limit, which includes continuity as a special case and which many concepts are based on, plays an important role in various branches of analysis (Artigue, 2000; Oktaviyanthi & Dahlan, 2018; Swinyard & Larsen, 2012; Winarso & Toheri, 2017). Despite this importance, it is seen that limit and continuity are among concepts which even students with high level of pre-learning have difficulty in giving meaning (Cottrill, Dubinsky, Nichols, Scwinngendorf, Thomas & Vidakovic, 1996; Juter, 2006; Przenioslo, 2004; Tall & Vinner, 1981). As a matter of fact, in these concepts, which are the basis for transition to advanced mathematical thinking, deficiency in learning causes weak algorithm operations in learning related to analysis. This situation is considered important for students who start taking advanced mathematics education and are expected to conceptualize limit and continuity in two ways such as dynamic (informal) and static (formal). Therefore, another subject that is as important as the limit-continuity concept appears: "limit-continuity teaching and the necessary method to be used in limit-continuity teaching." This shows the importance of the way of presenting learning-teaching activities in concretizing the limit-continuity subject, which has an abstract structure.

Obviously, when software with a dynamic feature is effectively used with an appropriate subject in mathematics teaching, many relations, features and generalizations that cannot be created in traditional environments can be studied easily. In this respect, it is thought that CAS environments provide students with an opportunity to develop their skills in interpreting symbols, working with symbols and reasoning, in short, abstract thinking. In addition, CAS environments, different from traditional environments, create a strengthening game environment that aims to develop students' visualizing, exploring and developing mathematical ideas. Briefly, it offers students the opportunity to make use of graphical and numerical representations along with symbolic representations (Laborde, 2001). Therefore, it is important to use software such as CAS depending on the use of technology in mathematics teachers' contemporary learning, teaching and evaluation processes. One of the problems that studies, as well as the present one, involving the use of CAS software were concerned with is how to develop students' abstract thinking skills regarding mathematical subjects in computer-aided learning (CAL) environments. Of course, here, the individual's understanding of mathematical concepts depends on the learning environment and his/her actions, and the teaching and concepts related to these actions do not have to be concrete. Shortly, learning environments involving the use of CAS software should provide students with the opportunity for abstract thinking about their own actions. For this reason, in the present study, the subject of limit-continuity taught in a learning environment in which the Derive software, one of CAS software, was used. The researcher teacher tried to provide the students with the opportunity to build their understanding on their own actions in this environment.

The effectiveness of mathematics course in the development of the features that guide logic and thinking is considered important in terms of professional development that should be gained by individuals. An individual with professional development competence is the one who thinks, learns and produces as well as demonstrates good performance thanks to his qualified workforce. The way to have a qualified workforce is possible with well-planned vocational education. In this sense, Vocational Schools (VS) of universities, which fulfill the function of filling the gap between Vocational and Technical Education institutions that give education at undergraduate and associate degree levels and the employment areas targeted by secondary education institutions, play an important role (Karadeniz & Kelleci, 2015; Karakuş, 2013). Vocational Schools were established to train intermediate staff equipped with sufficient knowledge and skills for the industry, commerce and service sectors. Thus, it is important to train qualified staff and to develop employees' abstract thinking

skills. In this respect, the importance of teaching and learning mathematics, which is known to improve thinking, becomes apparent. In line with this importance, considering the fact that 40% of students who have completed their secondary education and succeeded in the university placement exam in our country continue their education in associate degree programs, it is inevitable to teach the concepts included in the content of the general mathematics course taught in the vocational schools of our universities. However, this importance is not taken into account for the concepts of limit-continuity, in relation to which the conceptual understanding dimension is neglected due to the concern of preparing for the university placement exam, although they are included in the final year curriculum of secondary education institutions. For example; VS students perceiving infinity as a number think that an uncertain function can be continuous. In addition, VS students who do not examine the function graph until they graduate from secondary education have difficulty in adapting the formally-defined limit-continuity concepts to any problem they encounter. Obviously, students cannot adequately understand even the pieces of information they have memorized. This situation leads to the neglect of the dimension of conceptual understanding of the abstract concepts of mathematics and causes VS students to have memorized knowledge. In this respect, it is considered important to revise and evaluate the change in students' levels of learning in a CAS-aided learning environment designed for the concepts of limit-continuity which is taught in VS and whose dimension of conceptual understanding is neglected. In this study, which included students at different levels in relation to learning environments, it was considered appropriate to use the SOLO taxonomy (Structure of the Observed Learning Outcome) developed by Biggs and Collis (2014) in order to deeply evaluate the observed learning outcomes of students regarding limit-continuity.

The SOLO taxonomy is a taxonomy that is aimed at evaluating the cognitive knowledge and skills of students at different levels (Biggs & Collis, 2014). In addition, SOLO provides a hierarchical model for qualitative analysis of students' responses to specific tasks. It is seen that it is widely used as an effective tool in interpreting and evaluating the mathematical thinking skills of students in relation to certain concepts from primary education to university (Kabaca & Musan, 2014; Vallecillos & Mareno, 2002). Moreover, it could be stated that especially in studies whose purpose is to see the whole process and to measure the quality of the answers given by students, SOLO is an appropriate evaluation in terms of classifying students' thinking skills.

When the literature is examined, it is seen that there is not much room for preparing a technology-supported learning environment in VS students' learning the limit-continuity subjects and that the focus is generally on pure studies or misconceptions (Çeziktürk Kipel, 2013). In addition, it is seen that the experimental design is generally used to examine what VS students have learned about mathematical concepts and that the learning environments which the cases are connected to are interpreted with the support of qualitative data. Since the common purpose of these studies is to describe how much students have learned, the evaluations chosen for these studies might be reasonable. However, in this study, the main purpose of which was to understand how VS students learn, it was considered appropriate to choose the preferred model as the SOLO Taxonomy while evaluating and interpreting students' learning. In this way, while giving meaning to students' responses, the focus will be on finding answers to the questions of "what happened? What is happening? What will happen in the future? And How can we understand best?" as well as on what level of the SOLO taxonomy the learning outcomes correspond to.

When the thinking stages of the SOLO model are examined, it is seen that the abstract (formal) stage corresponds to the early adulthood period (Biggs & Collis, 2014). Since the interpretation of the limit-continuity concepts requires abstract thinking skills, it was assumed that the VS students, who are associate's degree students, were also in the abstract phase and were considered appropriate for this study to be carried out. In this respect, the study is important with its purpose of evaluating VS students' learning of limit-continuity with the SOLO taxonomy by using the Derive software from CAS in the CAL environment. Within the framework of this importance and the information presented above, this study tried to find answers to the problem of "What is the level of VS students' learning of 'the limit value of the function at a point and its image at that point' and 'thinking that continuity cannot be sought at the points where the function is undefined' in the CAS-supported environment according to the SOLO taxonomy?"

2. Method

2.1. Research Model

In this study, as the purpose was to examine the reflections of the teaching plan prepared within the scope of the limit-continuity example regarding the use of the Derive software from CAS upon VS students' learning, the study was carried out with the action research method, one of qualitative research methods. Action research is a systematic research process conducted by teacher researchers in order to evaluate the learning processes of their students in the learning/teaching environment (Mills, 2003). In addition, although action research is defined as a process to improve the quality of teaching, it provides researchers with the opportunity to work in their own classroom environment (Johnson, 2005). Therefore, in this study, where the practitioner was also the researcher, the researcher developed an action plan, conducted applications and evaluated the VS students' learning in the teaching process. In this respect, the study was designed in accordance with the action research model.

2.2. Study Group

The study group was made up of 32 students attending a Vocational School at a state university located in the east of Turkey. While determining the participants of the study, depending on the purpose of the study, the convenience sampling (Patton, 2014), which provides advantages in terms of accessibility, time and cost, was preferred. As a matter of fact, this study is considered to be suitable for convenience sampling in terms of being an action research in which the lecturer was the researcher and within the context of working with the students the researcher was teaching. In the study, the students involved in the process were coded as S1, S2, ... S32.

2.3. Data Collection Tools

In action research, data are collected through observation, interview, audio-recording and documentation (Philips & Carr, 2009). In this study, the data were collected with the help of a part of Derive-supported worksheets (taking the focused problems into account) prepared by Ertem Akbaş (2016), screenshots, observations, researcher teacher's notes, dialogues with students and audio-recording. Mutual confirmation of the data obtained with different methods allows increasing the validity and reliability of the results (Yıldırım & Şimşek, 2013). For this reason, the data collection tools specified in this study were used together. In addition, after the worksheets used within the scope of the study were re-evaluated by the researcher teacher, the content validity was obtained by taking the opinions of two experts. In addition to correcting the incomprehensible points in the worksheets arranged in line with the opinions of the experts, attention was paid to the fact that the students should discover the information and make generalizations by drawing conclusions with clue questions and without directly transferring the information to the students. Thus, it was thought that the mathematical thinking and abstraction capacity of the students would increase and that a more conceptual mathematics would emerge. The table below presents the target outcomes and strategic goals with the worksheets used in this study.

Table 1: Target Outcomes and Strategic Goals with Worksheets

Worksheets	Conceptual Clues	Target Outcomes	Strategic Goals
Worksheet-2	Function concept, factoring, ∞ concept, uncertainty states and limit concept	<ul style="list-style-type: none"> - Distinguishes the limit value of the function at a point and the image of the function at that point. - Finds the limit value of the function in cases of uncertainty - Examines the function graph 	<ul style="list-style-type: none"> - Explains the limit of trigonometric functions on the graph and conducts related applications - Explains the $\frac{0}{0}$ uncertainty state at the given point and calculates the limit of the function $\frac{0}{0}$
Worksheet-7	Concept of function, factorization, concept of limit and continuity	<ul style="list-style-type: none"> - Examines the graph of the function and finds the intervals in which it is continuous. 	<ul style="list-style-type: none"> - Determines whether the function is continuous or discontinuous at a given point and explains this on the graph

- Tells that continuity cannot be sought at points where the function is undefined.	- Explains the continuity of a function in an interval on a graph
- Establishes a relationship between limit and continuity	- Specifies the properties of continuous functions in the closed interval and explains them on the graph.

2.4. Data Analysis

Strauss (1987) emphasized that data analysis methods in qualitative research cannot be standardized and that standardizing data analysis will limit the qualitative researcher. In general, the most striking and common point in the recommendations regarding qualitative data analysis is the importance attached to the description of the data (Yıldırım & Şimşek, 2013). Taking this importance and the SOLO taxonomy into consideration, the qualitative data obtained were analyzed in three groups.

The Way Followed for Determining the Levels of Learning

The basic data for this study were those obtained from the learning outcomes of the students. In order to analyze the learning outcomes more easily, the questions in the worksheets were grouped within the framework of the research problem and analyzed separately. Below is the table related to grouping the questions in the worksheets used in this study:

Table 2: Grouping the Questions in the Worksheets

Name of the Question Group	Worksheet	Sub-Research Questions	Target Outcome of the Question Group
First Question Group	2	2, 3, 4, 5	Determines the limit value of the function at a point and the image of the function at that point and interprets the relationship between them
Second Question Group	7	2, 3	Says and comments that continuity cannot be sought where the function is undefined

The data obtained in line with this grouping were transformed into text and read several times in line with the research questions. The messages that each student tried to give were coded, and vignettes covering the problem situation were written (Baki, 1994). Following this, these vignettes were brought together for each question group and evaluated comparatively, and rubrics were created. Together with the researcher teacher, another researcher who had knowledge about the SOLO taxonomy took part in this process. The researcher teacher and the other researcher independently assigned the answers of the VS students to a level by using the rubrics. Thus, all the data were re-read and discussed with the other researcher, and an agreement was reached on the most appropriate level of thinking. The agreement percentage between the researchers was calculated with the reliability formula of Miles and Huberman (1994) and was found to be 82%. According to Miles and Huberman (1994), considering that a percentage of 70% and above is a reliable coding, we see in this study that the rubric developed based on the levels of the SOLO taxonomy was suitable for consistent and reliable leveling.

The Way Followed in Determining the Average Level of Learning

In the analysis process, for the calculations, the rubrics related to the SOLO taxonomy were classified with the help of a numerical scaling (1-5) (Mooney, 2002). In the rubrics, 1 shows an answer at the level of pre-structure (PS), 2 at the level of uni-structure (US), 3 at the level of multi-structure (MS), 4 at the level of relational structure (RS) and 5 at the level of abstracted structure (AS). These levels helped determine the VS students'

average levels of learning the subject of limit-continuity in a CAS-supported environment. Some answers were found to be below or above the classification even though they had properties related to the SOLO level. In this type of answers, the sign "-" was put in front of those below the level, and the sign "+" was put in front of those above the level. This showed a numerical increase or decrease of 0.25 point (Mooney, 2002).

Analysis of the Data Collected via the Dialogues

The researcher teacher was in constant dialogue with the students in order to evaluate their learning experiences in depth. In this process, the researcher teacher took explanatory notes and audio records when necessary. These notes and records were read and listened to over and over again, and the data required for the purpose of the study were supported with direct quotations and interpreted with the rubrics considering the levels of the SOLO taxonomy.

2.5. Application Process

Within the scope of the study, a total of 12-hour course flow was maintained with the VS students for four weeks. The researcher teacher, who intended to understand the students' learning about the subject of limit-continuity in depth, took on the role of researcher and lecturer during the 4-week study. In addition, in order for the students to use the "Derive" software effectively, a guide containing the menus in the software, the tools in the menus and the features of these tools was prepared. Table 3 summarizes the role of the researcher and the teaching process applied accordingly.

Table 3: The Teaching Process Applied and the Researcher's Role

Week	Researcher's Role	Applications Conducted	Application Process
Week 1	Explanatory and guiding	Conceptual clues about the subject of limit-continuity were given. The "Derive" introductory guide was distributed.	3-hour lesson process
Week 2	Explanatory and guiding	By introducing the software with the help of the guide, applications such as finding limits, drawing graph and continuity analysis were carried out with the students.	3-hour lesson process
Week 3	Observant, explanatory and guiding, providing consistency	The second worksheet (Worksheet-2) was applied with the help of the software.	3-hour lesson process
Week 4	Observant, explanatory and guiding, providing consistency	The seventh worksheet (Worksheet-7) was applied with the help of the software.	3-hour lesson process
TOTAL			12 hours

3. Findings

Within the scope of this study, the structure of the learning outcomes of the VS students in the CAS-supported learning environment was evaluated according to the SOLO taxonomy. In this section, the findings of the study are presented under two headings within the framework of the research problem. Under these headings, the descriptive analysis of the answers given by the students and the levels according to the SOLO taxonomy is presented together with the reasons. In the dialogues, the researcher teacher was coded as "RT," and the participating Vocational School students were coded as "S1, S2, ... S32".

Findings Related to Distinguishing the Limit Value of the Function at a Point and the Image of the Function at That Point

Under this heading, the answers given by the VS students within the scope of the “First Question Group” were revealed and examined.

In general, it was seen that the VS students had no difficulty in defining the given function. On the other hand, it was revealed through the dialogues that the VS students questioned whether the function was wrong or not, saying “the denominator becomes zero, and this function has probably been given incorrect” when they saw that the value of $f(x) = \frac{\sin(2x-2)}{3x-3}$ was $\frac{\sin(0)}{0}$ in $x=1$. According to VS students, the use of Derive software to find the limit of the function at a given point helped reach the limit value of the function at this point. Below are sample answers of the students related to this question group:

S7: Teacher, is there anything wrong with this function? Or I'm doing wrong.

RT: Why?

S7: Because when I write 1 instead of x in the function, the denominator becomes zero. You told us that when the denominator is zero, there is no solution set.

RT: So what to say about the function in such cases?

S7: It is an undefined function.

RT: If you cannot find the answer to the question asked in the first step, you might be interested in the other steps.

S7: Teacher, since this function is undefined, would it be correct to look at the other steps? (S7 was only doing paper and pencil work in this process.)

RT: I don't know, if you want, try to find the answers using the software as directed by the worksheets.

S7: All right.

The screenshot shows the following steps in the software:

- #1: $f(x) := \frac{\text{SIN}(2 \cdot x - 2)}{3 \cdot x - 3}$
- #2: $\lim_{x \rightarrow 1^-} f(x) := \frac{\text{SIN}(2 \cdot x - 2)}{3 \cdot x - 3}$
- #3: $f(1) := 2/3$
- #4: $\lim_{x \rightarrow 1^+} f(x) := \frac{\text{SIN}(2 \cdot x - 2)}{3 \cdot x - 3}$
- #5: $f(1) := 2/3$
- #6: $\lim_{x \rightarrow 1} f(x) := \frac{\text{SIN}(2 \cdot x - 2)}{3 \cdot x - 3}$
- #7: $f(1) := 2/3$

Figure 1: S7's operations on the software

S7: There is indeed a value. So writing down the given point was not the definition of limit.

RT: Then, what's the definition of limit?

S7: Even if limit is undefined at one point, it is that there are limits from the right and left at that point.

RT: What is limit on the right and left?

S7: it refers to taking the same values while approaching the limit from the right and from the left... Approaching the point is something different, and finding the value at the point is something else!

RT: So can you tell us what the difference between them is? I want you to think out loud.

S7: Actually, limit is something like a border, and when you approach that border, it is limit... So what's the value at the point? I am confused ... The value at that point is not limit, but when approaching from the right and left, it becomes limit. In fact, if it was up to me, I would not say that there is limit here, but you find a value when you make an operation in Derive!

S7 During this thinking process, he was looking at his work in Derive software and trying to gather the knowledge in his mind. The comments made by the student on the worksheet were as follows:

Şimdi de aynı fonksiyon için $x=1$ 'e iki taraftan da yaklaşırken alacak olduğu değeri yani $(\lim_{x \rightarrow 1} \frac{\sin(2x-2)}{3x-3})$ değerini bulmaya çalışınız. Ayrıca aşağıdaki tabloyu doldurup bulmuş olduğunuz değerleri birbiri ile kıyaslayarak nedenini açıklamaya çalışınız.

f(x) fonksiyonunun $x=1$ noktasındaki değeri	f(x) fonksiyonunun $x=1$ noktasına küçük değerlerle yaklaşırken aldığı değer	f(x) fonksiyonunun $x=1$ noktasına büyük değerlerle yaklaşırken aldığı değer	f(x) fonksiyonunun $x=1$ noktasına yaklaşırken aldığı limit değeri
$\frac{0}{0}$ belirsizlik	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$

1. $f(x) = \frac{\sin(2x-2)}{3x-3}$ fonksiyonuna $x=1$ değerini girdiğimizde $\frac{0}{0}$ belirsizliği bulduk.

2. $f(x) = \frac{\sin(2x-2)}{3x-3}$ fonksiyonunda $x=1$ noktasına küçük yani soldan yaklaştığımızda aldığı değer $\frac{2}{3}$ olarak buluyoruz. Aynı programda deneyerek emin olduğumu söyleyebiliriz.

3. $f(x) = \frac{\sin(2x-2)}{3x-3}$ fonksiyonuna büyük değerlerle yani sağdan yaklaştığımızda $\frac{2}{3}$ değerini bulduk programda deneyerek görsünü net bir sonuç ulaştırılır.

4. $f(x) = \frac{\sin(2x-2)}{3x-3}$ fonksiyonuna her iki taraftan sağlınsaldan $x \rightarrow 1$ yaklaştığımızda $\frac{2}{3}$ 'ü bulduk. Çünkü bir fonksiyonda soldan ve sağdan limitler eşitse her iki taraftan yaklaştığımız limitlerde eşitlik vardır.

Figure 2: S7's writing in the worksheet

When the answers given and written on the worksheet by S7 were examined, the idea of getting closer to the point $x = 1$ based on the student's knowledge of the number line was found striking. This thought of the student shows that he had knowledge of the subject. In addition, the biggest problem experienced by S7, who was aware that the value of the function at this point and the value it would take while approaching this point from small-large values were different, was that he failed to combine these pieces of knowledge. The value expected by the student as limit was that it would be the image at that point. In this process, although the Derive software helped the student find the value at the given point, the student was in doubt due to the uncertainty he obtained. The operations performed on the software by S7, who looked for a solution at the given point before finding the limit, are as follows.

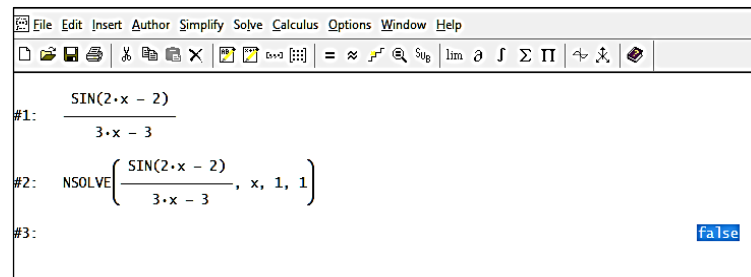


Figure 3: The solution made by S7 on the software before finding the limit

S7: Teacher, when we write the point in the given function, even the computer says "false." So it is wrong! I mean we can't find the limit, right?

At the very beginning of the questions, S7, who said that this expression did not have a limit value without doing any work on the limit of the function, failed to distinguish the limit value and the value of the function at a given point. In short, the student had difficulty in combining the pieces of knowledge. From this point of view, S7's answers were shaped around the uncertainty of the statement and placed at the "US" level because the focus was on only one aspect of the question. When all the answers were examined, expressions similar to the answers of S7 were encountered in general. These expressions are as follows;

"This point helps us here reach the uncertainty of $\frac{\sin(0)}{0}$."

"The denominator is zero, and this has no solution"

"there is uncertainty, but there is limit in the software"

"There is the right-left limit"

It was seen that besides S7, some other students (S11, S15, S17, S22 and S31) could not write the given point. In addition, these six students, who were not aware of what it means the function approaches a given point, could only find the limit value with the Derive software. They were also unable to come up with any idea of how to find the limit value they found. They also failed to establish a relationship between the limit value and the image of the function at that point. From this point of view, as it was seen that these students focused only on one aspect of the question being studied and that their answers regarding the question group were limited, these

answers were placed at the level of "US." As an example of this situation, S11's statement of "I cannot write the point instead" on the worksheet can be given;

$f(x)$ fonksiyonunun $x=1$ noktasındaki değeri	$f(x)$ fonksiyonunun $x=1$ noktasına küçük değerlerle yaklaşırken aldığı değer	$f(x)$ fonksiyonunun $x=1$ noktasına büyük değerlerle yaklaşırken aldığı değer	$f(x)$ fonksiyonunun $x=1$ noktasına yaklaşırken aldığı limit değeri
<i>değeri sıfır ya da yazamam</i>	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$

Figure 4: S11's writing in the worksheet

In general, when the VS students' dialogues with the researcher teacher, the worksheets and the screenshots were examined in depth, it was seen that the expected learning could not be achieved. Nevertheless, it was seen that the answers given by some students at the US level at the beginning turned into the MS level as a result of the completion of the activity by moving the same questions to the Derive environment. In fact, it was revealed that some of the thoughts of the VS students who, for the first time, encountered the CAL environment developed. The dialogue between S30 and RT is given below to exemplify such situations;

S30: Teacher, while approaching the value you gave, shouldn't I write that value instead of x in the function and solve it?

RT: Let's accept it as you said!

S30: So why did you ask again the value it will take for $x=1$ here?

RT: So let's ask like this; x approaching 1 and x being equal to 1, are these the same statements?

S30: Teacher, there is a sine here. Even if I write 1 where I see x , I cannot find the result! (Using paper and pencil, the student writes 1 where he sees x in the function.) The solution here is zero because the denominator is zero!

As no relationship had been established between the limit value and the image of the function at that point up to that time, it could be stated that the student thought at the "US" level. The dialogue between RT and S30, who brought the same questions to the Derive environment in the next period, was as follows:

RT: S30, you can use the software for the results you can't find.

S30: I found it "false" again, so wrong!

RT: Can you continue it by thinking aloud?

S30: "find limit" "limit point" Let's write 1. Result $\frac{2}{3}$!, now let's find for "left," "right." The same result $\frac{2}{3}$!

RT: What can you say in such a situation?

S30: The limit, right limit, left limit are all the same! Only the solution for $x = 1$ is "false"!

RT: So what can you say about this situation?

S30: Now even if there is no result at the point $x = 1$, the function has a limit. It also has a limit from the right and left.

As this dialogue shows that the answers given by S30 to the questions which he transferred to the software contained disconnected pieces of knowledge, it could be stated that the student thought at the level of "MS" in this process. In addition to the dialogues given as examples, the notes taken by the researcher teacher revealed that the CAS environment was encouraging for learning. Some of the notes taken by the researcher teacher are given as examples below:

"I was very curious about how to do the math lesson with the computer. This is great!"

"The software is very useful and easy, you can easily reach the result of any mathematical operation."

"Teacher, when you wrote on the blackboard, I was having my friend write and then I was taking it from him. But in this lesson, I tried to do something, too."

"It is very enjoyable to do operation with Derive. You write the function and find the limit."

"While we cannot write the given value where we see x in the function, we find limit in the software."

“Without the Drive, I would have been definitely unable to find the answers to the questions given in the worksheets.”

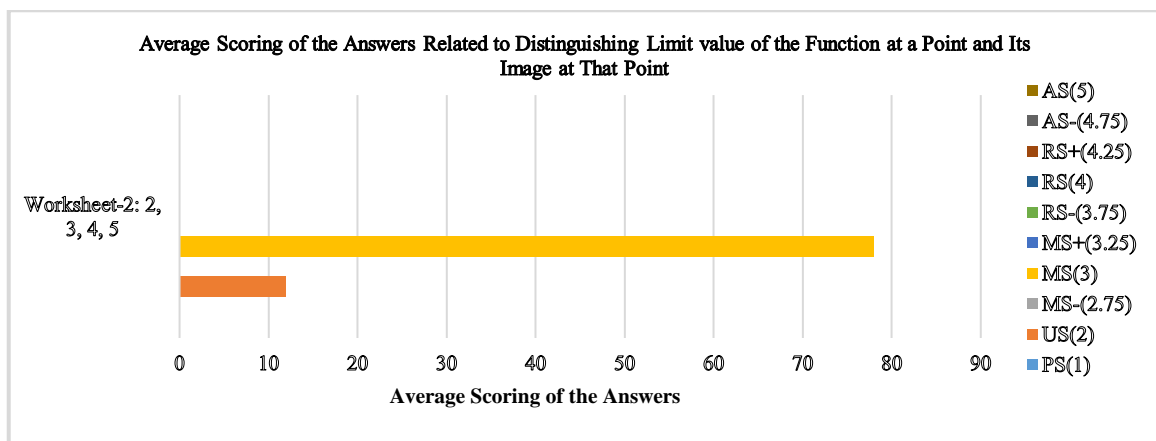
From the notes above, we see that the CAL environment was effective on the students’ participation in the lesson. In addition, it is quite clear that the Derive software used was remarkable and fun for the students. When the learning outcomes of the VS students in relation to this were examined, the answers given by some VS students at the US level at the beginning increased to the level of MS with the completion of the activity and with the dialogues. Thus, six of the student answers could be placed at the US level, and 26 at the MS level. The table below presents the SOLO leveling obtained from the answers of the VS students in relation to learning this outcome within the context of the number of students.

Table 4: Number of Student Answers According to SOLO Level

Name of the Question Group	Worksheet	Question Number	SOLO Levels														
			-	PS (1)	+	-	US (2)	+	-	MS (3)	+	-	RS (4)	+	-	AS (5)	+
First Question Group	2	2, 3, 4, 5					6			26							

Table 4 demonstrates that according to the students’ answers, 6 learning outcomes at the level of (US, 2) and 26 at the level of (MS, 3) were achieved. Thus, it could be stated that the answers given by the VS students in relation to distinguishing between the limit value of the function at a point and the image of the function at that point were generally at the level of (MS, 3). Accordingly, although the VS students draw one or more correct conclusions, they failed to associate them consistently with each other.

By examining the data given in Table 4 and the data analysis of these data, the average level of the answers given by the VS students regarding this outcome was determined. The average scoring of the answers corresponding to the SOLO levels is summarized in the graph below.



Graph 1: Average scoring of the responses corresponding to the SOLO levels

Graph 1 shows the average scoring of the answers given to the questions about distinguishing the limit value of the function at a point and its image at that point. According to this scoring, 6 (US, 2) corresponds to an average scoring of 12 (US), and 26 (MS, 3) to an average scoring of 78 (MS).

When the tables, graphs and dialogues given above were examined, it was seen that the students who turned to paper-pen before using the software gave answers focusing on one aspect of the questions and that the answers developed with the use of the software and contained independent and meaningful pieces of information. In this sense, the CAS environment could be said to contribute to the learning process of the VS students.

Findings Related to Thinking that Continuity Cannot be Required Where the Function is Undefined

Under this heading, the answers given by the VS students within the scope of the "Second Question Group" were revealed and analyzed.

In the questions in this group, the VS students were asked to find a solution for $f(x)=0$ after defining the given function $f: [-1, 2] \rightarrow \mathbb{R}, f(x) := x^2 + 2x$. Here, the function has a certain definition interval, and in order to explain the continuity in a certain interval without drawing a graph, it should be known that continuity cannot be sought at points where the function is undefined. Some of the students who thought that these questions could be solved easily with paper and pencil did not feel the need to use the software. The dialogue between RT and S29, who used the software to show the accuracy of the solutions he found, was as follows;

S29: Teacher, here, I was asked a solution for $f(x)=0$. Is this the solution set?

RT: Yes, S29, it is the solution set for $f(x)=0$ for this function.

S29: The question asked me to use the Derive software. Let's see if it will find 0 and -2 like me. (While S29 was saying this, he was doing the solution of the function in the worksheet and, at the same time, trying to find results in the software).

```

#1: f(x) := x*x + 2*x
#2: x*x + 2*x
#3: SOLVE(x*x + 2*x, x, Real)
#4: x = -2 v x = 0
#5: f(x) = 0
#6: SOLVE(f(x) = 0, x, Real)
#7: x = -2 v x = 0
  
```

Figure 5: Operations done by S29 on the software

S29: Super software! I can prove whether what I did is correct or not.

RT: You can move on to the next question.

S29: The question asks which one is in the definition interval and whether it cuts the x-axis? Why does it ask this? Wouldn't we be interested in continuity?

RT: Why do you think it is asking that?

S29: If there was an interval, we had to find the values in that interval. Then we were looking to see if it was less than zero or greater. Isn't that so, teacher?

RT: Go on.

S29: I didn't fully understand this and I do not know what I am doing it for. We did something like this. If there is a closed interval, we find the values of the limit points and multiply them ...

1) Fonksiyon $f(x)=0$ 'da driver programında yaptığımız
 #1! $f(x) = x^2 + 2 \cdot x$
 #2! $x^2 + 2 \cdot x = 0$
 #3! $\text{SOLVE}(x^2 + 2 \cdot x = 0, x, \text{Real})$ (çıktık)
 #4!
 $x = -2 \vee x = 0$
 sonucunu bulmuş ve fonksiyon $x=0$ 'da tanımlidir.
 2) 2. aşamada bulduğumuz gibi $f(x)$ fonksiyonu $x=0$ 'da tanımlidir.
 Bu aşamada $f(-1) = ?$ ve $f(2) = ?$ nedir bulmamız lazım ve
 $f(-1) \cdot f(2) < 0$ olduğunu bulmamız gerekir $f(x_0) = 0$ olduğunu bulmuş yarı
 $x=0$ 'da.
 $f(-1) = (-1)^2 + 2(-1)$ $f(2) = 2^2 + 2 \cdot 2$ $f(-1) \cdot f(2) < 0$
 $f(-1) = +1 - 2$ $f(2) = 4 + 4$ $-1 \cdot 8 < 0$
 $f(-1) = -1$ $f(2) = 8$ 'dir' $-8 < 0$ olarak bulmuş.
 # Fonksiyonu x eksenini $x=0$ noktasında keser ve $0 \in [-1, 2]$ 'dir'.

Figure 6: S29's writing in the worksheet

RT: What did we do these for?

S29: It says in the question examine the continuity ... If there is lack of definition in the closed interval, we have to see it so that we can find the continuity.

When S29's answers were examined, it was seen that in the advanced stages, the student had difficulty in the questions which he initially regarded as simple. As the answers of S29 were close to conceptual understanding, it is obvious that his answers were above MS. Despite this, S29, who did not conceptually combine the pieces of knowledge in his answers, could not give an adequate answer regarding the continuity of the function. S29's answers were placed at the level of "MS+" because they contained memorized pieces of knowledge far from conceptual understanding.

When all of the answers were examined, there were similar statements in the answers of S7, S9, S16, S20 and S28 similar to those of S29. Among the other students, the answers of S2, S3, S5, S18, S21, S23 and S32, who focused on the solution of function for zero, were placed at the level of "CY," although they tried to examine the continuity. The dialogue between S3 and RT is given below to exemplify the answers given at the MS level.

S3: To find the solution for $f(x)=0$, we set the function to zero. Then why would we try to find the point left in the domain? I didn't understand this!

```
#1: f(x) = x*x + 2*x
#2: x*x + 2*x
#3: SOLVE(x*x + 2*x, x, Real)
#4: x = -2 v x = 0
#5: f(x) = 0
#6: SOLVE(f(x) = 0, x, Real)
#7: x = -2 v x = 0
```

Figure 7: Operations done by S3 on the software

RT: You can associate it with the subject of our lesson.

S3: Continuity! Teacher, there are -2 and 0 in the real solution of x. Here we draw a graph to look at continuity. We look at these points. We understand whether it is continuous or not.

RT: So what could be the reason for giving an interval here? How would you associate it?

S3: An interval is given so that we can see if the function is continuous or not. We look at the interval of $[-1,2]$ in the graph, if we can draw without raising our hand, it is then continuous.

Although S3's answers to examine continuity in the closed interval were not sufficient, the fact that he found a solution for $f(x)=0$ and questioned this solution was effective in placing his answers at the level of "MS." Among the remaining students, the answers of 19 students who could not give information about how to examine continuity in closed interval but tried to find a solution for $f(x)=0$ were placed at the "US" level. Some of the students' statements are given below as an example of the answers at the US level:

"We can find $f(x)=0$ without using the Derive."

"Are we going to substitute zero for x here?"

"How do we find if it cuts the x-axis?"

"The worksheet we did in the previous lesson was better. Here we use Derive less ..."

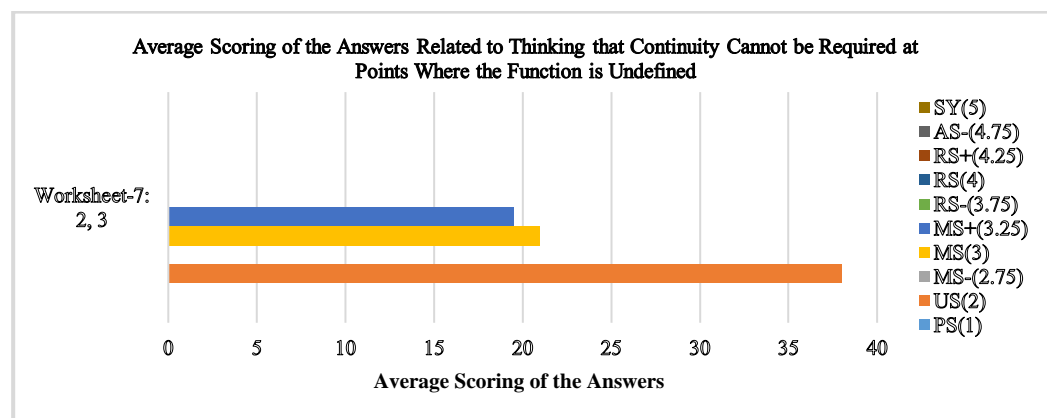
When all of the learning outcomes of the VS students regarding this outcome were examined, it was seen that 19 of the student answers could be placed at the level of US, 7 of them at the level of MS and 6 of them at the level of MS+. In the table below, the SOLO leveling obtained from the answers of the VS students in relation to learning this outcome is indicated within the context of the number of students.

Table 5: Number of Students' Answers According to the SOLO Level

Name of the Question Group	Worksheet	Question Number	SOLO Levels											
			PS (1)		US (2)		MS (3)		RS (4)		AS (5)			
Second Question Group	7	2, 3	-	+	-	+	-	+	-	+	-	+	-	+
			19				7		6					

Table 5 shows that among the students' answers, a total of 19 learning outcomes were obtained at the level of (US, 2), 7 at the level of (MS, 3) and 6 at the level of (MS+, 3.25) were obtained. Thus, it could be stated that in these questions examined in relation to thinking that continuity cannot be sought at points where the function is undefined, the answers given by the VS students were generally at the levels of (US, 2) and (MS, 3). According to this situation, the VS students focused only on one aspect of the question, failed to form conceptual understanding and could not associate the pieces of knowledge.

By examining the data given in Table 5 and the data analysis of these data, the average level of the answers of the VS students regarding this outcome was determined. The average scoring of the answers corresponding to SOLO levels is summarized in the graph below.



Graph 2: Average scoring of the answers corresponding to the SOLO levels

Graph 2 shows the average scoring of the answers given to the questions about being able to say and interpret that continuity cannot be sought at points where the function is undefined and interpreting it. According to this scoring, 19 (US, 2) corresponds to an average scoring of 38 (US); 7 (MS, 3) to an average scoring of 21 (MS) and 6 (MS, 3+) to an average scoring of 19.5 (MS+).

When the learning outcomes presented above to summarize this group were examined, it was seen that the answers containing independent pieces of memorized information developed with the use of the software. In this sense, CAS could be said to contribute to students' abstract thinking in the learning process.

4. Discussion and Conclusion

In the environment where CAS software was used, the learning outcomes of the VS students were evaluated as to be below the RS level according to the SOLO taxonomy. The use of CAS software allowed the VS students to increase the quality and structure of the learning outcomes at the level of PS and US to the level of MS and above. In line with this general result, in this section, the results will be discussed under two headings within the framework of the research problem.

Most of the answers given by the VS students about distinguishing the limit value of the function at a point and the image of the function at that point from each other improved to the level of MS. Although there were

answers below this level (US), no answer at the top level was encountered. These answers, which were not close to conceptual understanding, revealed that the expected learning did not occur and that VS students were not successful in integrating the meanings in a consistent structure in terms of abstract thinking skills. The expected learning did not occur, yet it could be stated thanks to the learning outcomes that the student expressions developed and that the learning environment contributed to this development. In the dialogues with the VS students who got accustomed to the use of the software, it was seen that as the questions progressed, the students turned to the computer in front of them, became more interested in mathematics by focusing on the lesson, had the curiosity of doing the mathematics lesson with a computer and would not be able to do the activities without the software. In addition, the CAS-supported learning environment was effective in helping the students question their memorized knowledge and abstract thinking. In the related literature, it is possible to meet studies on the effect of visual representations on learning by improving conceptual understanding and reasoning skills in CAL environments (Camacho, Depool & Santos-Trigo, 2010; Hutkemri, 2014; İlhan & Aslaner, 2020; Yıldız & Aktaş, 2015; Yorgancı, 2019). The common results of these studies cover not only the appropriateness of the software to be used for technology to have a positive effect on learning but also the importance of the activity contents to be prepared. Therefore, according to the findings of this study, although the software played a more effective role in the learning process of the students than the blackboard, the necessity of creating technology-supported content to support the learning-teaching process emerged. This necessity is similar to the research results reported by Sevimli and Delice (2015). With the help of the dialogues and her notes, the researcher teacher concluded that environments where visual elements are not used encourage students to memorize. In addition, the researcher teacher revealed that passing a class without sound concept knowledge and having only secondary school mathematics knowledge are not sufficient for VS students. In real life, in technical programs, students cannot fully understand the equivalents of the mathematics course in their professional life. The most important factors here are visualilty and interactive use. Thus, according to the researcher teacher, it is important to present the limit subject, which forms the basis of higher education mathematics, to the students with visual and conceptual relationships. This is especially important for the insufficient and memorization-based mathematics knowledge (US and MS-) that VS students take during their secondary school education. In the study, it was seen that during the lesson process, the students had difficulty in finding the limit by approaching the given point from small-large values and in distinguishing the limit of the function at a point and its image at that point, though included in the secondary school limit calculations. This situation is consistent with the result obtained by Biber and Argün (2015), who reported that students who start higher education without strong mathematical concept knowledge have inadequate mathematical knowledge, as well as with the result obtained by Aygün, Durukan, Aydın and Diril (2015), who showed that the most important share in school success differs depending on the students of the school and on the type of the secondary school graduated from.

In this study, in which the learning outcomes were evaluated with SOLO, the answers given by the VS students in relation to distinguishing the limit value of the function at a point from its image could not go beyond the levels of US and MS, which included operational and memorized pieces of knowledge. As a matter of fact, in the process of examining the limit of the functions at the given points in the question groups, the students tried to write the given point for x in line with their previous memorized knowledge. At the beginning of the study, the VS students, who had the misconception that the value of the function should exactly equal the limit value, were not aware of what reaching a value or convergence could be. It could be stated that the answers starting at the US level turned into the answers at the MS level. This situation revealed that the answers of the students who got used to the software and looked for limit on the software developed even though they did not integrate in a consistent structure. In addition, towards the end of the teaching process, it was seen that some of the VS students got rid of the misconception that the function's limit at one point could be equal to its image at that point.

Most of the answers given by the Vocational School students regarding thinking that continuity cannot be sought at points where the function is undefined remained at the level of US and MS. Most of the answers given by the VS students in relation to thinking that continuity cannot be sought at points where the function is undefined remained at the level of US and MS. The answers of the students who focused on the solution they found rather than on the continuity of the function, who stated that they would not be able to examine continuity without graph, who failed to associate their solutions with theoretical knowledge or failed to understand were placed at

the US level. As the study progressed, it was observed that the answers given developed from the US level to the MS level. Considering that the expected learning was at the RS level in the SOLO taxonomy, it is clearly seen that the VS students did not have this competency. In this sense, it was concluded that the VS students were not able to give sufficient information about discontinuity or to demonstrate the expected success. In addition, it was seen that the students had difficulty in what to do when the function was undefined. The reason for this might be the undefined function as well as the missing knowledge of the VS students about continuity. The results obtained in the related literature (Çeziktürk Kipel, 2013; Davis & Vinner, 1986; Tall & Vinner, 1981; Turan & Erdoğan, 2017) regarding the discontinuity knowledge support this conclusion. Especially the statements of the VS students who stated that "there is continuity where there is limit" without checking whether the function was defined or not were parallel to the misconception in the results obtained by Baştürk and Dönmez (2011).

It was seen that the VS students expressed continuity more clearly in their work on graph. Some of the VS students associated the graphs they drew on the software with real-life concrete shapes such as "slide, heartbeat, fairy chimney, arms that go to eternity"; made sense of convergence by moving on the graph and reaching the points where there was discontinuity; and gave meaning to infinity by making the graphic smaller and larger and examining the arms extending towards eternity. This situation revealed that the VS students who gave inconsistent answers were more successful in their works on the chart. As a result, it could be stated that the software brought comfort and concreteness to the virtual and tiring structure of mathematics. Derive software, which allowed visual presentation of mathematics, played a role in the development of the students' interest in mathematics and in shaping the learning outcomes. In this respect, the answers given could be said to be technology-oriented. The researcher teacher frequently saw that the VS students who drew graph on the software had misconceptions such as "if there is a gap in the graph, it is not defined, no limit, and not continuous" for. Accordingly, the researcher teacher noted the reason for this as the limit-continuity subject, uncertainties, infinity concept and function deficiency. Some of the studies that examined the limit-continuity subject and uncertainty situations and identified students' deficiencies reported similar results (Biber & Argün, 2015; Dane, Çetin, Bas & Sağırlı, 2016; Elia, Gagatsis, Panaoura, Zachariades & Zoulinaki, 2009; Sierpiska, 1987; Szydlik, 2000). The researcher teacher took notes emphasizing that traditional teaching methods are not sufficient to overcome these deficiencies; that learning environments should be created to support students' learning; and that especially environments where visual elements are not used encourage students to memorize. In the literature on technology-aided mathematics education, studies indicating that visual representations are effective in teaching complex subjects that students have difficulty understanding (Aztekin, 2012; Ertem Akbaş, 2019; Hutkemri, 2014; Sevimli & Delice, 2015; Yıldız & Aktaş, 2015) support the findings obtained by the researcher teacher in the present study. According to the researcher teacher, presenting the subject of limit-continuity, which is the basis for higher education mathematics, to students with visual and conceptual relationships is important in developing students' abstract thinking skills about these concepts. This is especially important for VS students' inadequate and memorized mathematics knowledge. The fact that the students who stated that they took mathematical or non-mathematical courses throughout their vocational secondary school education and that they never studied on limit-continuity is an indication of this importance. These statements, which were at the US level focusing on one side of the question, were interpreted as another effect of the students' failure to express that continuity could not be sought by examining the points where the function was undefined.

However, it was seen that the answers given by the VS students with the graph they drew on the Derive software were more meaningful. Thus, it could be stated that some answers initially considered to be appropriate to the US level included statements at the MS level as the study progressed. The visuals and technology in the dynamic environment appear to have a supportive role in the development of students' answers. In addition, it was concluded that the use of CAS was effective in the VS students' questioning their memorized knowledge about the strategies they followed for solution. This result reveals that the actions taken by the students in the software provided them with dynamic problem solving strategies.

In Turkey as well as in the world, in teaching the science of mathematics, which is subjected to students' comments such as "difficult, impossible to understand," new methods and software suitable for the technology of the era and today's student profile should be taken into consideration by teachers in mathematics teaching. It will

be beneficial to use such mathematical software in the development of learning outcomes of VS students who cannot fully comprehend the equivalent of mathematics especially in their professional lives.

References

- Artigue, M. (2000). Teaching and learning calculus: What can be learnt from education research and curricular changes in France? *CBMS Issues in Mathematics Education*, 8, 1-15.
- Aygün, M., Durukan, S., Aydın, İ., & Diril, H. Z. (2015). Meslek yüksekokulu matematik müfredatı ile DGS soruları arasındaki korelasyon. *Journal of Research in Education and Teaching*, 4(3), 289-292.
- Aztekin, S. (2012). Determining the understandings about the limit subject in mathematics by using repertory grid technique. *International Online Journal of Educational Sciences*, 4(3), 659-671.
- Baki, A. (1994). *Breaking with tradition: A study of Turkish student teachers' experiences within a Logo-based mathematical environment*. (Doctoral dissertation). University of London, London.
- Baştürk, S., & Dönmez, G. (2011). Mathematics student teachers' misconceptions on the limit and continuity concepts. *Necatibey Eğitim Fakültesi Elektronik Fen ve Matematik Eğitimi Dergisi*, 5(1), 225-249.
- Biber, A. Ç., & Argün, Z. (2015). The relations between concept knowledge related to the limits concepts in one and two variables functions of mathematics teachers candidates. *Bartın University Journal of Faculty of Education*, 4(2), 501-515.
- Biggs, J. B., & Collis, K. F. (2014). *Evaluation the quality of learning: the SOLO taxonomy (structure of the observed learning outcome)*. New Jersey: Academic Press.
- Camacho, M., Depool, R., & Santos-Trigo, M. (2010). Students' use of Derive software in comprehending and making sense of definite integral and area concepts. *Issues in Mathematics Education*, 16, 29-61.
- Cottrill, J., Dubinsky, E., Nichols, D., Schwinnendorf, K., Thomas, K., & Vidakovic, D. (1996). Understanding the limit concept: Beginning with a coordinated process schema. *Journal of Mathematical Behavior*, 15, 167-192.
- Çeziktürk Kipel, Ö. (2013). Meslek yüksekokulunda limit, türev, integral konuları üzerine bir vaka araştırması. *Journal of Education and Humanities: Theory and Practice*, 4(7), 13-26.
- Dane, A., Çetin, Ö. F., Bas, F., & Sağırılı, M. Ö. (2016). A conceptual and procedural research on the hierarchical structure of mathematics emerging in the minds of university students: An example of limit-continuity-integral-derivative. *International Journal of Higher Education*, 5(2), 82.
- Davis, R. B., & Vinner, S. (1986). The notion of limit: Some seemingly unavoidable misconception stages. *The Journal of Mathematical Behavior*, 5(3), 281-303.
- Elia, I., Gagatsis, A., Panaoura, A., Zachariades, T., & Zoulinaki, F. (2009). Geometric and algebraic approaches in the concept of "limit" and the impact of the "didactic contract". *International Journal of Science and Mathematics Education*, 7(4), 765-790.
- Ertem Akbaş, E. (2016). *Meslek yüksekokulu öğrencilerinin bilgisayar destekli ortamda "limit-süreklilik" konusundaki öğrenmelerinin SOLO taksonomisine göre değerlendirilmesi* (Yayınlanmamış doktora tezi). Karadeniz Teknik Üniversitesi, Eğitim Bilimleri Enstitüsü, Trabzon.
- Ertem Akbaş, E. (2019). The impact of EBA (Educational Informatics Network) assisted mathematics teaching in 5th grade fractions on students' achievements. *Journal of Computer and Education Research*, 7(13), 120-145.
- Hazzan, O., & Goldenberg, E. P. (1997). Students' understanding of the notion of function in dynamic geometry environments. *International Journal of Computers for Mathematical Learning*, 1(3), 263-291.
- Hutkemri, E. Z. (2014). Impact of using GeoGebra on students' conceptual and procedural knowledge of limit function. *Mediterranean Journal of Social Sciences*, 5(23), 873.
- İlhan, A., & Aslaner, R. (2020). Cabri ve GeoGebra yazılımları kullanımının, matematik öğretmen adaylarının geometrik şekiller üzerine akıl yürütme becerisine etkisi. *Journal of Computer and Education Research*, 8(16), 386-403.
- Johnson, A. P. (2005). *A short guide to action research* (2nd edition). Boston: Pearson Education.
- Juter, K. (2006). Students' attitudes to mathematics and performance in limits of functions. *Mathematics Education Research Journal*, 17(2), 91-110.
- Kabaca, T., & Musan, M. S. (2014). The effect of dynamic mathematics learning environment on the SOLO understanding levels for equations and inequalities of 8th graders. *Mustafa Kemal Üniversitesi Sosyal Bilimler Enstitüsü Dergisi*, 11(26), 195-207.
- Karadeniz, M. H., & Kelleci, D. (2015). Meslek yüksekokulu öğrencilerinin matematik dersine ilişkin tutumlarının başarıya etkisi. *Karadeniz Sosyal Bilimler Dergisi*, 7(14), 1-16.
- Karakuş, C. (2013). Meslek yüksekokulu öğrencilerinin yaşam boyu öğrenme yeterlikleri. *Eğitim ve Öğretim Araştırmaları Dergisi*, 2(3), 26-35.

- Laborde, C. (2001). Intergration of technology in the design of geometry tasks with Cabri-Geometry. *International Journal of Computers for Mathematical Learning*, 6, 283-317.
- Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook*. London: Sage.
- Mills, G. E. (2003). *Action research a guide for the teacher researcher*. (2nd. edition). New Jersey: Pearson Education.
- Mooney, E. S. (2002). A framework for characterizing middle school students' statistical thinking. *Mathematical Thinking and Learning*, 4(1), 23-63.
- Oktaviyanthi, R., & Dahlan, J. A. (2018). Developing guided worksheet for cognitive apprenticeship approach in teaching formal definition of the limit of a function. *IOP Conf. Series: Materials Science and Engineering*, 335, 1-5.
- Papert, S. (1993). *The children's machine: Rethinking school in the age of the computer*. New York, USA: Basic Books, Inc.
- Patton, M. Q. (2014). *Qualitative research & evaluation methods: Integrating theory and practice*. London: Sage Publication.
- Philips, K. D., & Carr, K. (2009). Dilemmas of trustworthiness in preservice teacher action research. *Action Research*, 7(2), 207-226.
- Pierce, R., & Stacey, K. C. (2002). Algebraic insight: The algebra needed to use computer algebra systems. *The Mathematics Teacher*, 95(8), 622-627.
- Powers, R., & Blubaugh, W. (2005). Technology in mathematics education: Preparing teachers for the future. *Contemporary Issues in Technology and Teacher Education*, 5(3), 254-270.
- Przenioslo, M. (2004). Images of the limit of function formed in the course of mathematical studies at the university. *Educational Studies in Mathematics*, 55, 103-132.
- Renshaw, C. E., & Taylor, H. A. (2000). The educational effectiveness of computer-based instruction. *Computers & Geosciences*, 26(6), 677-682.
- Sevimli, E., & Delice, A. (2015). Can technology-assisted instruction improve theoretical awareness? The case of fundamental theorem of calculus. *Turkish Journal of Computer and Mathematics Education*, 6(1), 68-92.
- Sierpińska, A. (1987). Humanities students and epistemological obstacles related to limits. *Educational Studies in Mathematics*, 18(4), 371-397.
- Strauss, A. L. (1987). *Qualitative analysis for social scientists*. Cambridge: Cambridge University Press.
- Swinyard, C., & Larsen, S. (2012). Coming to understand the formal definition of limit: Insights gained from engaging students in reinvention. *Journal for Research in Mathematics Education*, 43(4), 465-493.
- Szydlík, J. (2000). Mathematics beliefs and conceptual understanding of limit of function. *Journal Research in Mathematics Education*, 31(3), 258-276.
- Tabuk, M. (2019). Lisansüstü tezlerde bilgisayar destekli matematik öğretimi uygulamaları: Meta-sentez çalışması. *Kuramsal Eğitimbilim Dergisi*, 12(2), 656-677.
- Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, 12, 151-169.
- Turan, S. B., & Erdoğan, A. (2017). Matematik öğretmen adaylarının "süreklilik" ile ilgili kavramsal yapıları. *Journal of Research in Education and Teaching*, 6(1), 397-410.
- Vallecillos, A., & Moreno, A. (2002). Framework for instruction and assessment on elementary inferential statistics thinking. *Teaching of Mathematics*, 7, 1-6.
- Wiest, L. R. (2001). The role of computers in mathematics teaching and learning. *Computers in the Schools*, 17(1-2), 41-55.
- Winarso, W., & Toheri, T. (2017). A case study of misconceptions students in the learning of mathematics: The concept limit function in high school. *Jurnal Riset Pendidikan Matematika*, 4(1), 120-127.
- Yıldırım, A., & Şimşek, H. (2013). *Sosyal bilimlerde nitel araştırma yöntemleri*. (Genişletilmiş 9. Baskı). Ankara: Seçkin Yayıncılık.
- Yıldız, Z., & Aktaş, M. (2015). The effect of computer assisted instruction on achievement and attitude of primary school students. *International Online Journal of Educational Sciences*, 7(1), 97-109.
- Yorgancı, S. (2019). Bilgisayar destekli soyut cebir öğretiminin başarıya ve matematiğe karşı tutuma etkisi: ISETL Örneği. *Türk Bilgisayar ve Matematik Eğitimi Dergisi*, 10(1), 260-289.