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Reconstruction of Five-Fold Patterns with the Concept of Key Length

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Abstract
Art, architecture, music and literature, which are thought to be far from mathematics, contain complex mathematics. The fact that Anatolian lands hosted many civilizations throughout history provided cultural richness including the relationship between mathematics and art and created Anatolian culture. Geometric patterns are frequently encountered in artistic and architectural works in Anatolian culture. Geometric patterns existing in Anatolian culture vary according to their fold. In this research, five-fold patterns have been explained mathematically by introducing the 'Key Length' concept. We developed a system that reconstructs an existing Islamic Geometric Pattern in the context of the girih mode that we presented verbally and visually in eleven steps. In addition, we introduced to the literature the concept of 'Key Length' which is used in the drawing process.

Keywords: Art-Culture-Maths, Geometrical Drawing, Girih Mode, Islamic Geometric Pattern, Key Length

1. Introduction

This section discusses Islamic geometric patterns and their properties in the literature (§ 1.1), then explains symmetry and angle properties of Islamic geometric patterns (§ 1.2), examines the girih mode and properties (§ 1.3) and finally explains five-fold patterns and the girih mode (§ 1.4).

1.1 Islamic Geometric Patterns and their properties

Geometric patterns are cultural constructions. Different cultures have produced different geometric patterns in history and many geometric patterns have survived to the present day as cultural symbols. It is possible to see broken lines, zigzags, and lozenges in the works of ancient Greeks and Sumerians. However, geometric patterns gained another identity with Islamic culture (Bonner, 2017; Demiriz, 2017; Necipoğlu, 1995). Although the archaeological evidence does not offer specific information on where and when geometric pattern drawing methods were first utilized, research suggests that geometric patterns were invented in Baghdad, the Abbasid capital (Aljanabi, 2019; Bonner, 2017; Necipoğlu, 1995). Many Western researchers also base the basis of geometric patterns on the Abbasids and the use of Arabic letters in calligraphy by developing the geometry and connecting them to a system (Abas & Salman, 2007; Şen, 2013). The geometric patterns that began to spread throughout the Islamic world were associated with a few new artistic and architectural forms; thus, they became a determinant and symbolic language. Islamic Geometric Patterns have three essential properties. These properties are that the patterns are symmetrical and constructed according to orders and rules, can be extended forever, and are anonymous (attribution of works to the architect of the period, artists not known by name) (Demiriz, 2017; Majewski, 2017).

In this study, we examined, explained, and used mathematics in the background rather than the historical development of Islamic Geometric Patterns. The next section describes symmetry and angle systems and the mathematical properties of patterns in detail.

1.2 Symmetry and angle properties of Islamic Geometric Patterns

Taking into consideration architectural constructions and historical records, numerous characteristics separate geometric patterns from one another. Despite this, a few design characteristics have been found (Aljanabi, 2019; Bonner, 2017; Majewski, 2019a) . The most apparent common mathematical property of Islamic Geometric Patterns is that their preparation process is in a similar symmetry axis system. The three primary symmetry axes used in Islamic Geometric Patterns are quadruple, quintuple, and hexadecimal systems. In other words, Islamic Geometric Patterns are four-, five-, six-fold, and all different patterns can be produced by utilizing the characteristics of four-, five-, and six-fold patterns (Aljanabi, 2019). The pattern fold can be briefly defined as the number of pieces separated when the corner angle (right angle) of the pattern's square or rectangular outer mold is divided into equal parts with the help of lines. The concept of pattern fold is critical as it also determines the angles of all polygons in the pattern.

The polygons forming the patterns with identical folds are related to each other. In other words, since the angle systems of the polygons are interconnected, the polygons complement each other (Aljanabi, 2019). Angles of 22.50 in four-fold patterns, 180 in five-fold patterns, and 150 in six-fold patterns remain between the lines dividing the right angle and form the angle system. In Figure 1, the angle systems used in the drawing of four-, five-, and six-fold patterns are shown in detail.

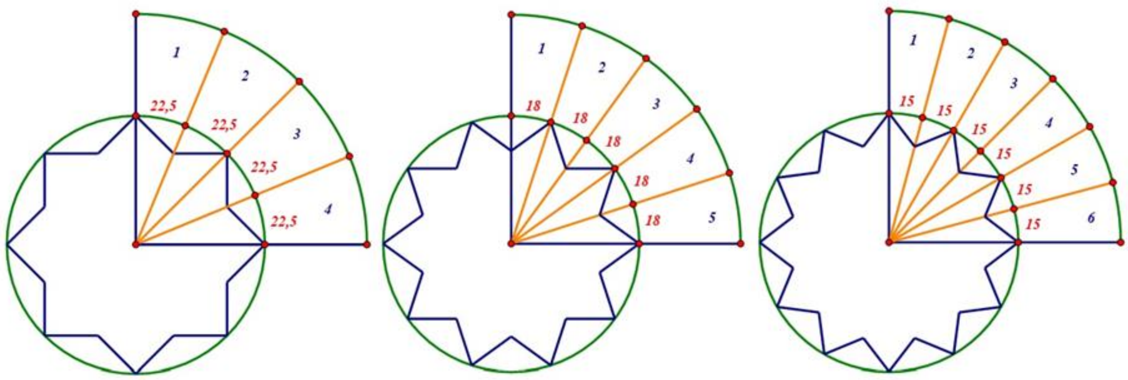


Figure 1: Angle systems of four-, five-, and six-fold patterns, respectively (Aljanabi, 2019).

Islamic Geometric Patterns have many common properties. However, closer inspection reveals that the patterns differ geometrically and contain various geometric components. This situation diversifies the techniques of constructing patterns. In many countries where Islamic culture is shared, local methods are used while constructing geometric patterns. In addition, there are some approaches to construct geometric patterns in Europe (Majewski, 2019a). In the next section, the girih mode, which is one of the approaches to constructing Islamic Geometric Patterns is explained in detail.

1.3 The girih mode and properties

The approach employed in the production of a geometric design determines the aesthetic character of the pattern (Bonner, 2017). No recorded text or book from that period describes how the Middle Ages Islamic Geometric Patterns came to be. (Bonner, 2017; Majewski, 2019a; Necipoğlu, 1995). Nevertheless, one was not completely unaware of that period's geometric pattern construction process (Majewski, 2017; Majewski, 2019a; Şen, 2013). Islamic Geometric Patterns often appear in many architectural structures, especially mosques. In addition, it has been a guide in constructing geometric patterns on drawing parchments belonging to architects living in the Middle Ages and later (Şen, 2013; Majewski, 2019a, Necipoğlu, 1995). Islamic Geometric Patterns reached their peak with the girih mode, which is the most widely used in the historical process (Lu, 2007; Şen, 2013). The most convincing proof that the primary historical method of constructing Islamic Geometric Patterns is the girih mode is the Topkapı Parchment (Majewski, 2019a, Majewski, 2019b; Bonner, 2017; Lu, 2007; Necipoğlu, 1995). Traditionally, Central Asian builders have described the way used in patchwork scrolls as girih (Persian, "knot")(Majewski, 2019b). This term refers to the knots or corners of network-like geometric grid systems used when constructing patterns for two- and three-dimensional architectural plans and decorative coverings (Necipoğlu, 1995).

Within the scope of the research, Islamic Geometric Patterns were examined and reconstructed in the context of the girih mode, based in Central Asia. The most important reason for choosing the mode is using geometric properties of polygons such as angle, side, and symmetry axis in the pattern construction process. Another reason for this choose is that using geometric elements in the drawing process makes the process simpler and more understandable (Necipoğlu, 1995). The flexibility level of the design methodology and related design diversity is also crucial in choosing the technique (Bonner, 2017). In addition, the prediction about girih mode usage on the parchments with patterns in the Topkapı Palace played an essential role in choosing the method (Bonner, 2017; Majewski, 2019a, Necipoğlu, 1995). All drawing steps of the patterns constructed using the girih mode are based on a geometric structure. The investigations revealed that geometric patterns were constructed under some established guidelines. Girih mode consists of contour, tessellation and motif.

The examinations determined that the geometric patterns were constructed according to some standard rules while being constructed with the girih mode. These rules are given below.
1. The polygons that make up the tessellation are convex and symmetrical.
2. The sides of the polygons that make up the tessellation are coincident.
3. The tessellation polygons are either located inside the frame or intersect symmetrically at the sides.
4. If a pattern line touches the side of any tessellation polygon, that side has another line running in the mirror image on the other side.
5. When two lines of the pattern touch the side of any polygon in the tessellation, these two lines continue inside the other polygon with a standard side without changing direction. Note: the angles between the pattern lines and the tessellation sides are equal.
6. When the two lines of the motif in the tessellation , the lines are bent.
7. The motif lines can only stand on the side of the frame.
8. Each motif has the same symmetry as the tessellation polygon it is in. (Majewski, 2019a; Majewski, 2017)

1.4 Five-fold patterns and the girih mode

Five-fold patterns are fundamental in terms of Islamic art and architecture history. In the literature, five-fold patterns, also known as decagon patterns, are the most popular patterns in Iran and Turkey (Bonner, 2017; Majewski, 2019a; Necipoğlu, 1995). Seljuk and Ottoman designers have produced many designs and interesting methods using five-fold patterns (Bonner, 2017; Cromwell, 2009; Demiriz, 2017; Majewski, 2019a; Majewski, 2019b). Angle properties differ in the outer frame properties, tessellation polygons, and motifs described in the title ‘Symmetry and Angle Properties of Islamic Geometric Patterns.’ For this reason, in the next section, the outer frame properties of the five-fold patterns and the tessellation polygons are explained in detail.

1.4.1 Contour properties and drawings of five-fold patterns

The contour can be square, triangle, hexagonal or rectangular. Sometimes a pattern can be built using more than one pattern. The contour of the pattern may vary depending on the surface to be covered (Majewski, 2019b). However, when the architectural structures belonging to Turkish-Islamic Culture are examined, it is seen that many of the patterns are squares or rectangles with particular proportions like squares.

In the context of the girih mode, it is crucial to divide the right angle into five equal angles in the rectangular contour drawing of five-fold patterns. Therefore, an [AB] of any length is drawn.

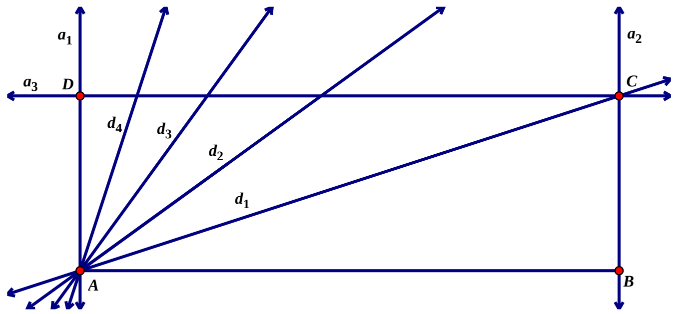


Figure 2: Rectangular contour drawing of five-fold patterns

The line a_1 passing through the point A and perpendicular to [AB] is drawn ($a_1 \perp$ AB). The resulting right angle (angle A) is divided into five equal angles. The lines dividing the angle are d_1, d_2, d3, and d4. The line a_2 passing through the point B and perpendicular to [AB] is drawn (a2 ⊥ AB). Since the rectangle is in the form C (1, 5), the point C, ion of a2 and d1 becomes the third corner of the rectangle. Let a3 the parallel line to [AB] passing through the point C. The point D, the ion of a3 and a1 becomes the fourth corner of the rectangle. [AB]-[BC]-[DC]-[DA] are drawn. All lines used in the drawing are deleted except the sides of the rectangle to complete the rectangular contour in the form of C (1, 5) (Figure 2). The ion point of a2 and the lines d1, d2, d3 and d4 determined the form of the rectangular countour respectively C(1, 5), C(2, 5), C(3, 5) and C(4, 5). The concept of tessellation, which differentiates the girih mode, is examined after the contour drawing.

1.4.2. Properties of polygons forming the tessellation of five-fold patterns

The tessellation is the overlay of polygons with coindicent sides, which are drawn inside the contour or the determined ground, facilitating the placement of motifs. The tessellation is deleted when the pattern drawing finishes. In other words, the hidden network allowing to draw the pattern can be defined as tessellation. The polygons that make up the tessellation of five-fold patterns are formed by breaking up the regular decagon differently. Ten frequently used polygons are shown in Figure 3. The angles of each of these polygons are multiples of 180. The tessellation of a pattern may contain all or several of the polygons in Figure 3. These polygons, differently named in the literature, were renamed by us in the study. Explanations on naming polygons are given in the second part of the study.

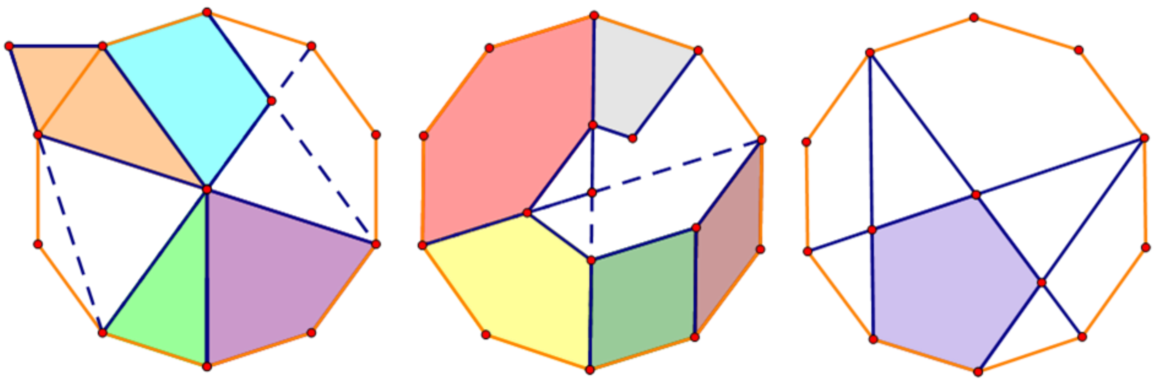


Figure 3: Polygons used in the tessellation drawing of five-fold patterns (Majewski, 2019a)

2. Contribution of the study to the literature

This section situates the importance of the study (§ 2.1), introduce the concept of Key Length (§ 2.2), explain the gradual system developed by the researchers for the use of the girih mode (§ 2.3), gives the names of the tessellation polygons of five-fold patterns (§ 2.4) and lastly presets the naming the motifs placed in the tessellation polygons in five-fold patterns (§ 2.5).

2.1. The importance of the study

Islamic Geometric Patterns were generally researched by architects and art historians in the literature. In this context, it can be said that the mathematical properties of the patterns have not been studied much. In this study, we examined in detail geometric patterns' mathematical properties. Many different geometric properties are used while creating Islamic Geometric Patterns with the girih mode. We haven't found any study in the literature that contains sufficient mathematical explanations about where and how to start the construction of the tessellation, which forms the framework of the girih mode, and how to continue it. In this study we explained mathematical properties to construct Islamic Geometric Patterns. The length required for the construction of the tessellation, i.e. the coincident sides of the tessellation polygons, is named as the 'Key Length'. We show mathematically how to obtain 'Key Length'in an empty frame. In addition, the reconstruction process of an existing Islamic Geometric Pattern in the context of the girih mode determined by us is explained in detail in eleven steps in total under the titles of analysis and reconstruction. In other words, the girih mode has become more systematic.

2.2 The concept of Key Length

We introduce the concept of Key Length, emphasizing the girih mode's mathematical properties and explaining where and how to use mathematics. The concept of tessellation is fundamental in patterns constructed using the girih mode. When the literature is examined, a lack of concept for the mathematical explanation of the tessellation drawing attracted attention. Obtaining the concept of Key Length using mathematics shows that the pattern drawings are not random but based on scientific foundations. The radius of the circle may be used to shift lengths, and equal sides can be readily constructed by determining this length. The concept was named 'Key Length' by the researchers of the study, based on the metaphor of quickly opening unlocked doors. The determination of the Key Length differs according to the pattern. Mathematical properties of angle, bisector, median, midpoint, diagonal and circle are frequently used to reach this length.

2.3 The gradual system developed for the use of the girih mode

There are research about the girih mode in the literature. However, these research is complex for interested people. We have considered and developed a gradual system to eliminate this deficiency and use it more easily. The developed system was based on the concept of 'Key Length,' and the system was composed of eleven steps under two titles. The two main steps of the system are analyzing and reconstructing the pattern. To reconstruct the pattern, it has to be analyzed mathematically. For this reason, we determined five crucial properties of Islamic Geometric Patterns as criteria for mathematical analysis. After the pattern analysis according to the given steps, we determined six steps to reconstruct the pattern. Next section is about the main steps of the system of analyzing and reconstructing the pattern.

2.3.1 Analysis of an existing five-fold Islamic Geometric Pattern

To construct an existing five-fold Islamic Geometric Pattern, a mathematical analysis of the pattern is required initially. Pattern analysis consists of five steps. These steps are sequential,

1. Determining the small pattern mold,
2. Determining the 'contour' properties of the small pattern mold,
3. Drawing tessellation on the determined pattern,
4. Determining the motifs in the tessellation polygons,
5. Determining the 'Key Length' required for the drawing.

2.3.1.1. Determining the small pattern mold. The first step of the pattern analysis is to determine the pattern's 'small pattern mold.' Small pattern mold can be defined as the smallest piece obtained by using the symmetrical property of Islamic Geometric Patterns and allowing to reach the whole pattern by repeating. The small pattern mold is determined by using symmetry axes, and there are no horizontal and vertical symmetry axes in this pattern. All drawing steps of the pattern are performed in these mold dimensions.

2.3.1.2. Determining the 'contour' properties of the small pattern mold. The small pattern molds of the five-fold patterns generally consist of rectangles. Most of the rectangles mentioned above have a special ratio between their sides. These rectangles are obtained by dividing the corner angle of the rectangular contour by five. The contour may consist of one or more rectangles depending on the pattern. In some patterns, the contour of the small pattern mold may not be well defined because it does not consist of particular rectangles. Rectangular contours generally follow this rule; although there are different drawing methods for molds that do not, this article is limited to the contours that comply with this rule.

2.3.1.3. Drawing tessellation on the determined pattern. The midpoints of the regular polygons that make up the pattern and the combination of these points can be used while drawing the tessellation on the determined pattern. Since the research is limited to five-fold patterns, the polygons frequently used while constructing the tessellation are presented within the scope of the research. Looking at the pattern, it is determined how to place these polygons in the rectangular contour with their sides coincident. In addition, eight rules of the girih mode should be considered while drawing the tessellation.

2.3.1.4. Determining the motifs in the tessellation polygons. The motifs in the tessellation vary according to the polygons. The motifs placed inside the polygons can be produced in any number without ignoring the angle and side properties. Although the motifs have different names in the literature, we renamed them using star names in this study by taking expert opinions considering the close relationship between astronomy and geometry for centuries.

2.3.1.5. Determining the Key Length of the pattern. The Key Length of the pattern is the length of the coincident sides of the polygons that forms the tessellation. For this reason, the Key Lenght varies according to the pattern and tessellation. It can be more than one, depending on the pattern.

2.3.2. Reconstructing an existing five-fold Islamic Geometric Pattern

After analyzing the pattern, the drawing step starts pattern design should consider eight rules of the girih mode. Considering these rules, the pattern is reconstructed or constructed in six sequential steps. These steps are,

1. Drawing the contour of the small pattern mold,
2. Obtaining the Key Length mathematically,
3. Construction of tessellation,
4. Drawing motifs inside the tessellation,
5. Checking the small pattern mold,
6. Completion of the pattern.

2.3.2.1. Drawing the contour of the small pattern mold. Rectangles with special ratios between their sides are used frequently in girih mode. The drawing steps of the five-fold patterns rectangles are explained verbally and visually in detail in the section of 'Contour Properties and Drawings of Five-Fold Patterns.'

2.3.2.2. Obtaining the Key Length. The Key Length differs from pattern to pattern as stated in the analysis step. Without the concept of Key Length, the explanations about the tessellation drawing will be insufficient mathematically. While obtaining the Key Length, the ion points of the lines dividing the right angle by five with the contour, the line segment lengths, the diagonal length of the contour, the side properties of the tessellation polygons, and the eight rules of the girih mode are used.

2.3.2.3. Construction of tessellation. Drawing the length is simple by utilizing the parallelism and perpendicularity characteristics of the lines and shifting the length with the Key Lengths.

2.3.2.4. Drawing motifs inside the tessellation. The motifs differ according to the polygons in the tessellation. Angle and side properties of tessellation polygons should be used considering the pattern fold while placing motifs inside them. Parallelisms, perpendicularities, bisectors, and the midpoints of tessellation polygons are used to place motifs.

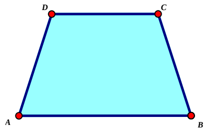
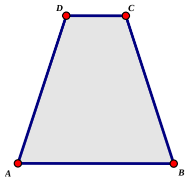
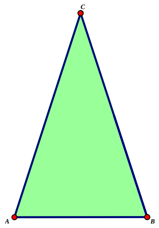
2.3.2.5. Checking the small pattern mold. Tessellation and motifs in the small pattern mold should be controlled according to the fold variable of the pattern. Since the research is limited to five-fold patterns, all angles of the polygons in the mold must be multiples of 18.

2.3.2.6. Completion of the pattern. The tessellation inside the small pattern mold is deleted to complete the pattern. Thus, only the motifs remain in the small pattern mold, and the pattern is ready to be reflected. The entire pattern is constructed by mirroring the small pattern mold as much as necessary (considering the axes of symmetry).

2.4. Naming the tessellation polygons of five-fold patterns

While naming the polygons that form the tessellation, 3-digit coding was performed. The first digit of the code consists of the initial letter of the known name of the polygon. The second digit consists of the numerical representation of the number of sides of the polygon. The third digit of the code consists of the letters A, B, C, and D to show the difference of polygons with equal sides. Examples of naming polygons are seen in Table 1.

Table 1: Naming the polygons

Polygon	Name of Polygon	Number of Sides of a Polygon	Polygon Form	Code Name of Polygon
	Trapezoid	4	A	**T4A**
	Trapezoid	4	B	**T4B**
	Triangle	3	A	**T3A**

2.5. Naming the motifs placed in the tessellation polygons in five-fold patterns

One of the most striking properties of Islamic Geometric Patterns is that they contain symmetrical shapes resembling 'stars' and 'constellations.' They have emerged from the Kufic art and gained different forms in the historical process. Star shapes appear in Islamic Geometric Patterns because astronomy and geometry are intertwined in Islamic culture. Many ancient mathematicians were also interested in astronomy and developed mathematical formulas about the positions of the planets. From the 9th to the 15th centuries, astronomy was the most passionate intellectual activity in the Islamic world. For this reason, the oldest observatories were built in Islamic geography, and Muslim astronomers firstly named many stars (Abas & Salman, 2007).

In the next section, the pattern of a window shutter of the Istanbul Üsküdar Mihrimah Sultan Mosque, which is the Architect Sinan work, was reconstructed using the concept of 'Key Length' and the sequential system of the girih mode. With the given an example, our contributions to the literature are revealed more clearly.

3. The process of reconstructing the pattern of the window shutter of the Üsküdar Mihrimah Sultan Mosque with the concept of Key Length.

This section gives the analysis the pattern of the window shutter of the Üsküdar Mihrimah Sultan Mosque with the concept of Key Lengt (§ 3.1) and explains the reconstruction of the pattern (§ 3.2).

3.1. Analysis of the pattern

This pattern is a five-fold pattern examined in detail within the scope of the research. It is possible to understand that the pattern is five-fold from the decagonal rosette in the middle or the quarter-decagonal rosette in its corners (Figure 4). Similarly, the regular pentagon is included among the polygons that make up the pattern, indicating that it has five folds.

Figure 4: Five-fold pattern determining

3.1.1 Determining the small pattern mold

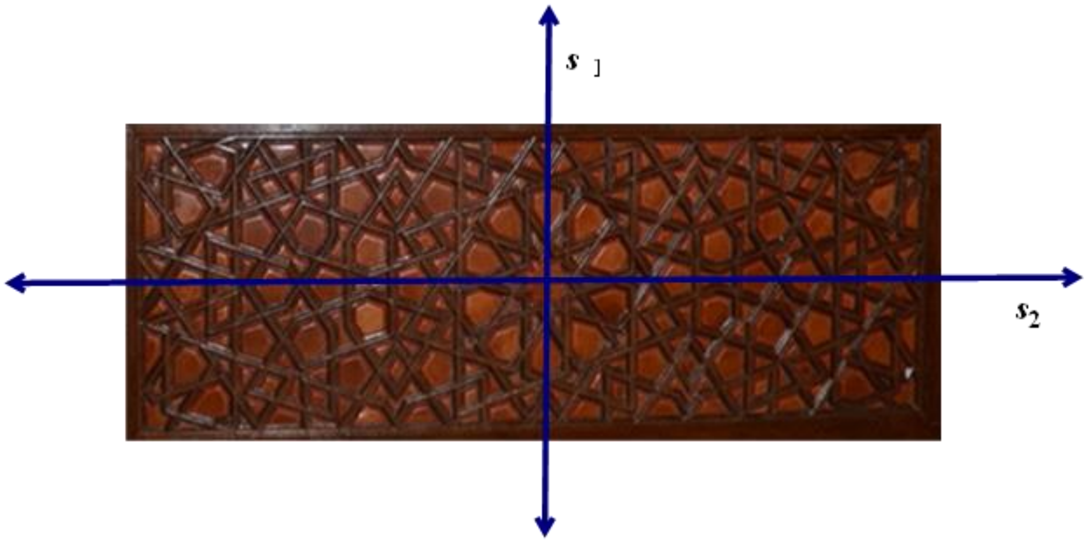


Figure 5: Symmetry axes of the pattern

A small pattern mold is found by using the symmetry axes. As seen in Figure 5, the small pattern mold of this pattern consists of a ¼ part of the whole figure. In other words, this pattern has two symmetry axes. The determined small pattern molds contain the same mathematical properties as they are symmetrical. For this reason, any of the four sections indicated in Figure 5 can be employed for the reconstructing and analysis procedure. In this study, all the design operations occur on the 1 in 4 pieces in the lower-left corner (Figure 6).

Figure 6: Small pattern mold belonging to the pattern

3.1.2. Determining the 'contour' properties of the small pattern mold

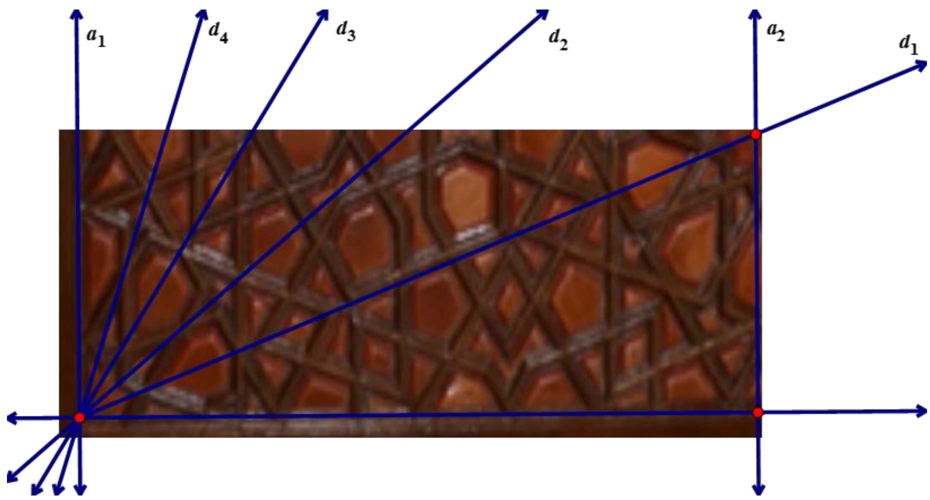


Figure 7: Contour size of small pattern mold

When the diagonal of the rectangular pattern is drawn, the diagonal passes through the corners or sides of the polygons making up the pattern. Line d1 dividing the right angle by five in the lower-left corner of the rectangle is seen in Figure 7 that it is the diagonal of the small pattern mold. For this reason, the contour of the small pattern mold obtained by using the symmetry axes is in the form of C (1, 5).

3.1.3. Determining tessellation on the small pattern mold

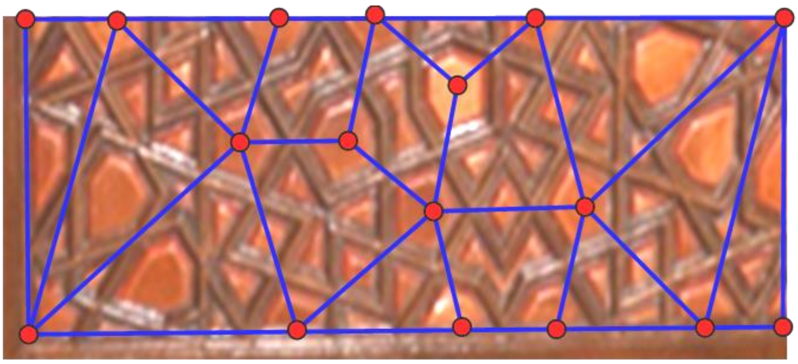


Figure 8: Tessellation drawing on the pattern

The tessellation of the pattern is drawn, as seen in Figure 8. The eight rules of the girih mode are considered while drawing the tessellation. In addition, the midpoints of the regular polygons forming the pattern are used. As seen in Figure 8, the tessellation of this pattern consists of four polygons. These polygons are named E4A - T3A - D4B - T4A. The highlighted and colored version of the tessellation polygons is shown in Figure 9.

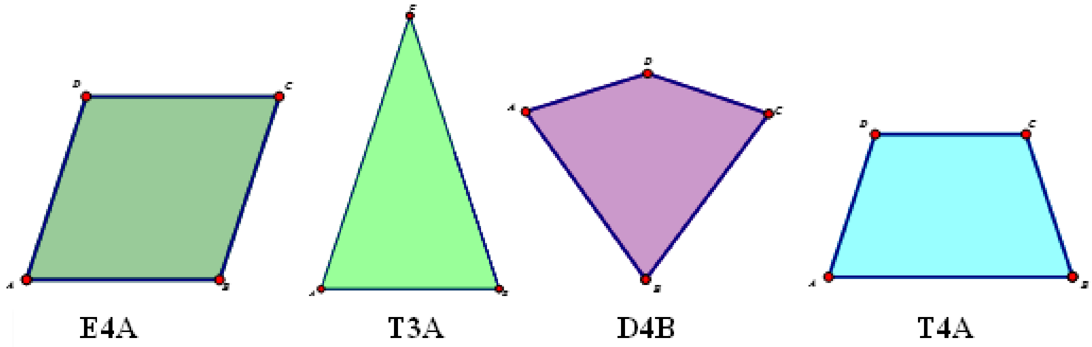


Figure 9: Tessellation polygons of the pattern

3.1.4. Determining the motifs in the tessellation polygons

The motifs situated inside the polygons are named after star names. These names are presented in Table 2.

Table 2: Motifs in the tessellation

Tessellation Polygon	E4A	T3A	D4B	T4A
Motif	*Zuhal*	*Rami*	*Anka*	*Mirza*

The highlighted and colored versions of the motifs in the tessellation polygons are shown in Figure 10.

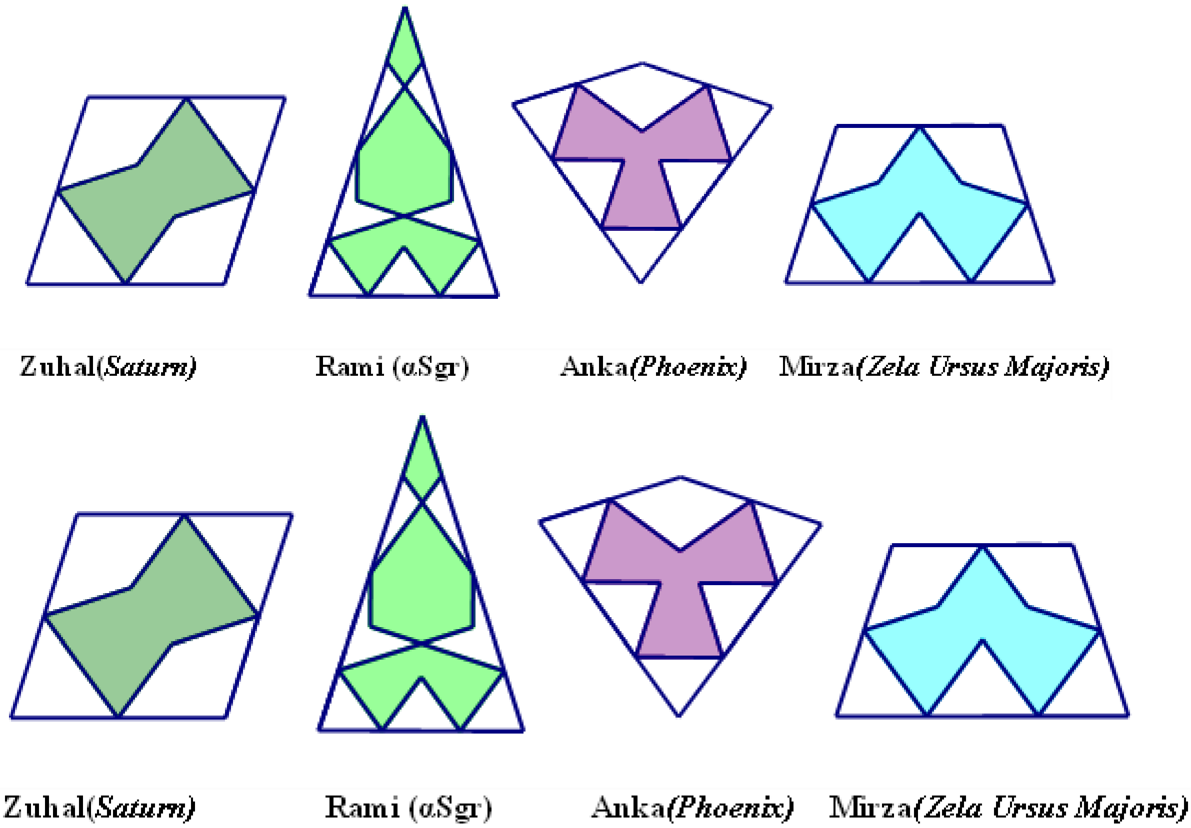


Figure 10: Pattern motifs

3.1.5. Determining the Key Length of the pattern

The short side length of T3A in the tessellation is equal to the long side of D4B and T4A. Also, the short side length of D4B is equal to the short side length of T4A and the side length of E4A (cf. Figure 9). In this context, the short side of T3A and the side length of E4A are the Key Lengths required for drawing.
Thus, the five-step pattern analysis finishes. The pattern has been analyzed and made easy to understand mathematically. The analyzed pattern has passed to the reconstruction step.

3.2. Reconstructing the pattern

As in the analysis step, drawing processes are carried out in the determined small pattern mold to avoid time loss and make the pattern easier to construct. The drawings included in the research were constructed in the Geometers' Sketchpad Program.

3.2.1. Constructing the contour of the small pattern mold

In the analysis phase of the pattern, it was determined that the contour was in the form of C (1, 5). The determined rectangle is explained with detailed visuals and verbal expressions under the section of 'Rectangular properties and drawings of five-fold patterns.' In Figure 12, there is a rectangle in the form of C (1, 5).

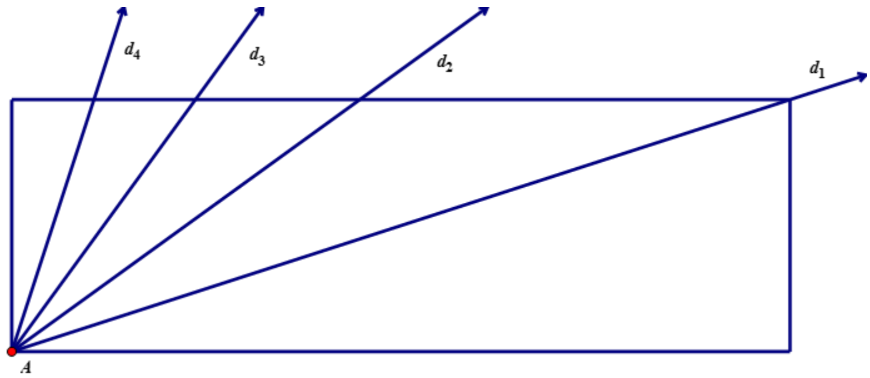


Figure 11: C(1, 5) Rectangle

3.2.2. Obtaining the Key Length

The Key Length required for the drawing has been identified during the analysis phase to be the short side length of T3A and the side length of E4A. To obtain this length, the draft tessellation is first carefully examined. Angle A is divisible by five, and the lines dividing the angle by five are not deleted. E is the point where the line d4 dividing angle A by five intersects the [DC] side. The length [AE] is the long side of T3A. C1 circle with center A and radius [AE] is drawn [C(A, AE)]. F is the point where the C1 circle intersects the line d2 [EF] is the short side of T3A. In other words, one of the Key Lengths is the length [EF]. Half D4B is located above the T3A in the corner. C2 circle with radius [EF] is drawn to construct D4B [C(A, AD)]. G is the point where C2 circle intersects the [DC] side. The length [FG] is the short side of D4B, the side length of E4A. So, the second Key Length is [FG]. The Key Lengths are in red, as shown in Figure 13.

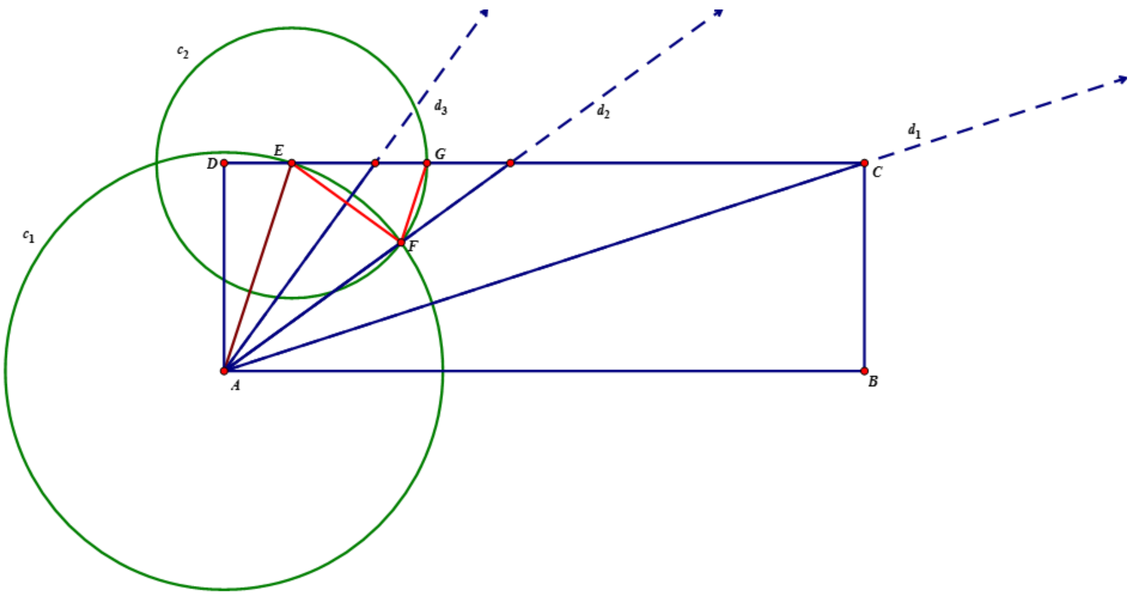


Figure 12: Key Length drawing of the pattern

3.2.3. Construction of tessellation

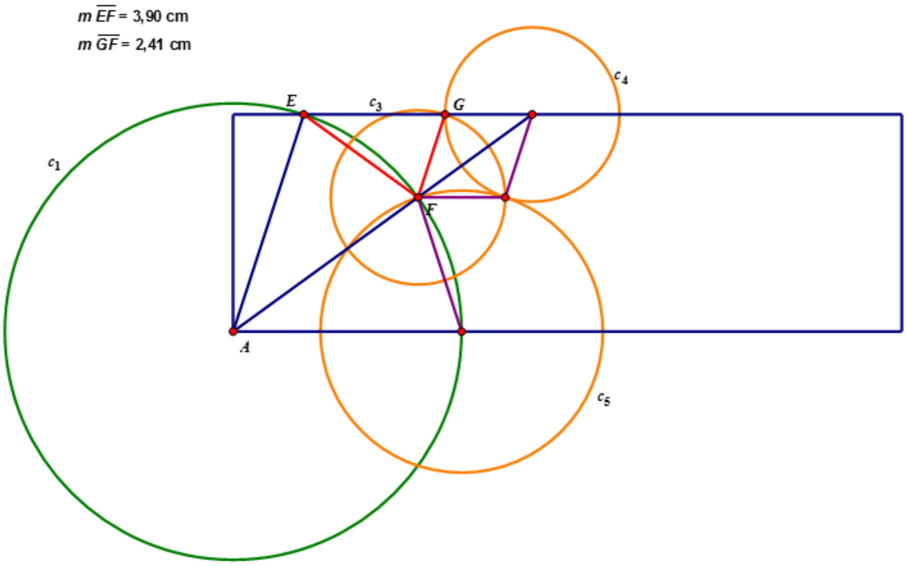


Figure 13a: Step 1

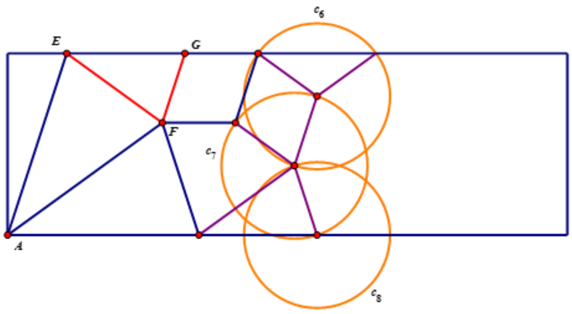


Figure 13b: Step 2

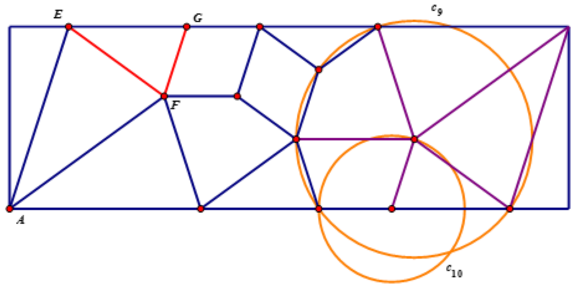


Figure 13c: Step 3

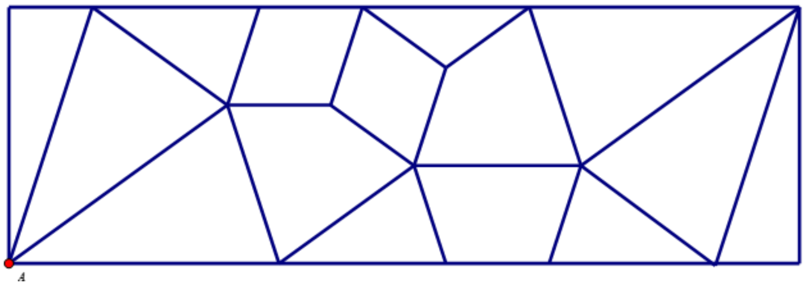


Figure 13d: Step 4

Figure 13: Steps of tessellation drawing of the pattern

The short side of T3A and the side length of E4A, the Key Lengths, were obtained by drawing. The tessellation was completed according to the draft analysis using the Key Lengths. While completing the tessellation, radius of circles, parallelism, and perpendicularity of the lines (Figure 14).

3.2.4. Drawing motifs inside the tessellation

The tessellation of the pattern consists of polygons E4A, T3A, D4B, and T4A. The motifs to be placed inside these polygons are Zuhal, Rami, Anka, and Mirza (Figure 15a and Figure 15b). The line continuity required for the completion of the pattern is provided by at least three corners of the motifs touching the tessellation polygons. The points where the motifs touch the tessellation polygon are usually the midpoint of the polygon's side. Each motif can be drawn separately inside the polygons of the tessellation, considering all the properties of the five-fold patterns. Another alternative is to draw any motif and to complete the motifs with the help of parallel and perpendicular lines, taking into account the rule of continuity of the lines. In this study, motif drawings are not explained in detail.

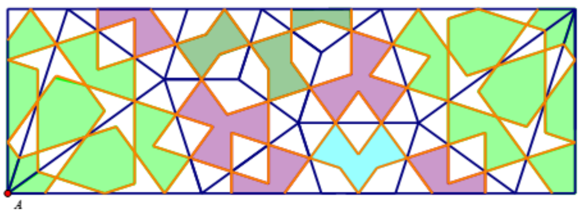


Figure 14a: Highlighted pattern motifs

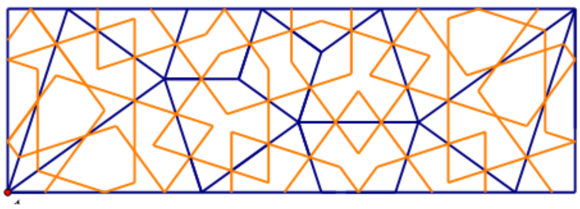


Figure 14b: Drawing of pattern’s motifs

3.2.5. Checking the small pattern mold

Since the pattern has five-folds, the angles of each polygon of tessellation and the motifs in the small pattern mold should be multiples of 180. Angles were measured in the drawing program, and each polygon properties were provided.

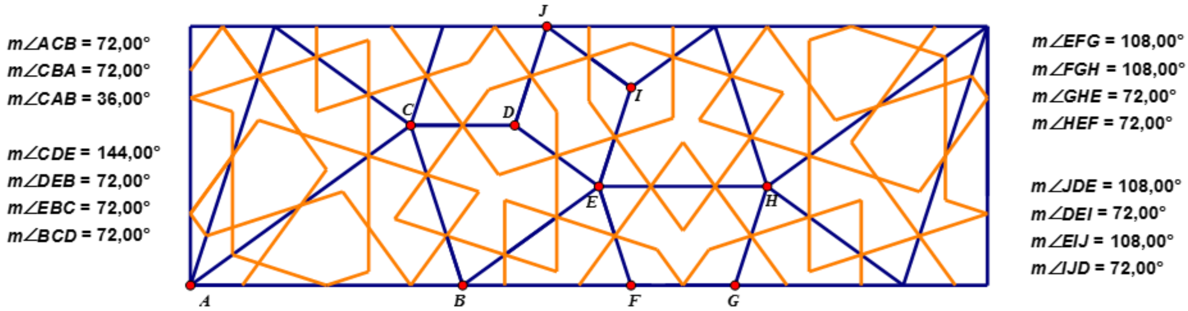


Figure 15: Angle properties of the polygons

3.2.6. Completion of the pattern

The tessellation of the small pattern mold, checked to complete the pattern, was deleted (Figure 17). Thus, only the motifs remained in the small pattern mold, and the pattern was made ready to be reflected.

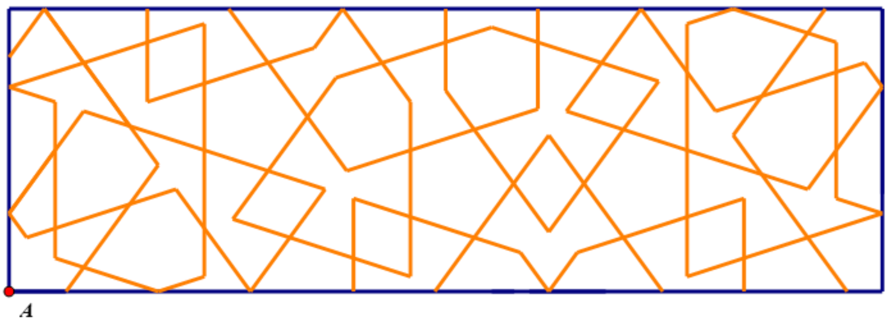


Figure 16: Small pattern mold

Since the small pattern mold of this pattern is 1/4 of the whole, the whole pattern was constructed by two mirroring operations with respect to [XY] and [ZZ’] (Figure 18a ve Figure 18b). Then the axes of symmetry were deleted. The final step of the pattern is in Figure 19.

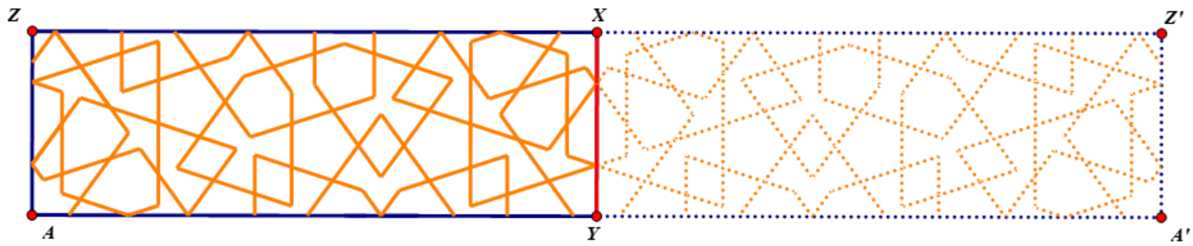


Figure 17a: Mirroring operations with respect to [XY]

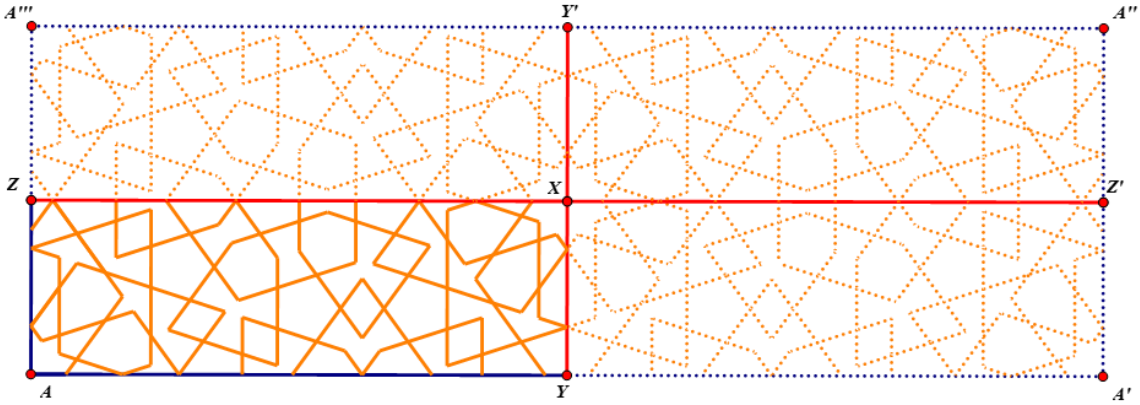


Figure 17b: Mirroring operations with respect to [ZZ']

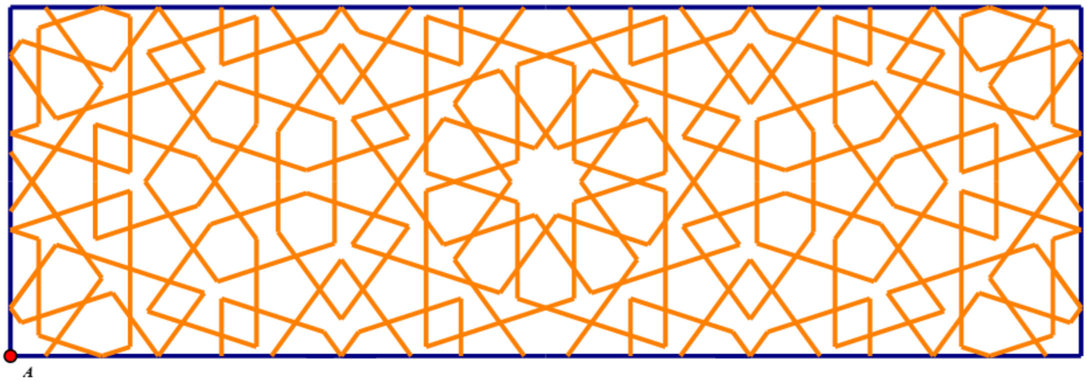


Figure 18: Üsküdar Mihrimah Sultan Mosque Window Shutter Pattern

4. Conclusion

It is not possible to completely abstract the mathematics that exists in life from culture. For this reason, it is very important to make sense of mathematical knowledge in the context of culture. As in other cultures, Anatolian culture was blended and fed mathematics with art. Geometric patterns are special structures where Anatolian culture blends mathematics and art. In the study, the concept of Key Length in the construction of geometric patterns provided the meaning of the knowledge. In addition, geometric patterns construct with the concept of Key Length are demonstrated. On the other hand, the polygons that make up the patterns that exist in the context of a culture are presented by naming them in the context of the same culture. It has been emphasized that these namings are much more important when it comes to a structure in which culture, art and mathematics progress together.

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