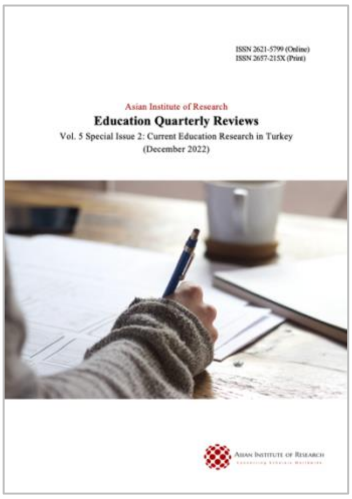

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Examination of the Concept Images of Pre-service Teachers for Single-Variable and Multi-Variable

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Abstract
This study aims to examine the concept images of pre-service mathematics teachers for the concepts of single-variable and multivariable functions. In the present research, which was conducted with the case study method, a test consisting of three open-ended questions was used to obtain data about the definition of the concept of function and the concept images of the single and multivariate functions of the pre-service teachers. The data obtained were coded with the descriptive analysis method and analyzed qualitatively. Each coding was given as a theme with frequency and percentage values and the answers given by the pre-service teachers were presented as examples. The findings obtained in this study showed that the pre-service teachers' concept images about function consisted of different representations of the concept of a function and concept images were generally classified as matching, transformation and equation. The pre-service teachers' concept images of the concept of function also had incomplete learning and misconceptions. That definition and value sets, which are important components of the concept of function, were not included in the answers. The pre-service teachers defined the functions as f(x) in multivariate function examples, and they ignored that x represents the only variable in the function. In addition, pre-service teachers who had misconceptions in single-variable function examples (equation and algebraic expression) made the same mistake in multivariable functions and could not make a correct representation although they used two variables in their expressions. It was determined that misinformation and misconceptions in concept images for multivariable functions were caused by incomplete information and mislearning in the function definition and single-variable function examples. The findings suggest that supporting the course designs on single and multivariable functions with examples and concept knowledge would be beneficial for an effective learning process.

Keywords: Concept Image, Single Variable Function, Multivariable Function

1. Introduction

Mathematical concepts are interrelated like links in a chain. Failure to establish a relationship between these concepts during mathematics teaching makes learning mathematical concepts challenging and causes students to develop a negative attitude toward mathematics. Therefore, effective learning of mathematical concepts is significant for both effective mathematics teaching and the development of positive attitudes toward mathematics. There are different theories about the process of understanding and learning concepts, one of which is the theory of concept definition and concept image, founded by Tall and Vinner (1981). In general terms, concept definition is the words and contents used to distinguish a concept from others. Concept image is all the content and associations that exist in the individual's mind regarding that concept. Vinner (1983) states that the existence of a concept in an individual's mind consists of two different cells: concept definition and a concept image. These cells are seen as mental structures that become active in actions related to the concept in question.

The mental image of a concept consists of visual representations, such as symbols, graphics, shapes, mental pictures and thoughts that appear in the person's mind in relation to that concept. All these mental thoughts and representations in the individual's mind regarding the concept constitute the concept image. Therefore, concept image is a phenomenon that can vary from person to person. Individuals have concept images that are right, wrong, incomplete or contradictory, which are specially created according to their own experience and thinking. Thus, concept image has an informal structure that has the potential to reflect the misconceptions and wrong views of the person toward that concept and the the concept image of the person does not have to be compatible with the formal definition of the concept as it is personal (Rösken & Rolka, 2007). Concept definition, on the other hand, is expressed as a scientifically accepted definition. In other words, the formal definition represents the concept definition.

In light of the studies on concept images and concept definitions, it has been determined that students tend to use the concept of image instead of using concept definitions. Although there are concept definitions and concept images of some concepts in mind, these two may not always coexist for every concept; because, for some concepts, there is only the concept image in mind and there is no concept definition. In other words, while students are solving a problem, they can establish an interaction between these two and use only concept definition or concept image (Vinner, 2002). For example, a student can use only the concept image for the function without the need for a formal definition of the function. In such cases, there is no need for a formal definition of the concept, as the concepts are expressed based on the concept images acquired in the past years (Tall & Vinner, 1981).

When it comes to effective learning, meaningful interaction between concept definition and concept image is a prerequisite. The interaction between concept image and concept definition can occur in two different ways (Vinner, 1983). (i) In the first case, when a student is introduced to the concept, he/she first creates a mental image, and then the teacher gives the definition for the concept. After that, there are three different situations. In the first, the student assimilates the concept definition given by the teacher and changes the initial concept image. In the second, the student keeps his/her own concept image, changes the definition given by the teacher, and creates a mental definition that resembles his/her own concept image. In the third, the concept definition and concept image of the student remains the same. When the student is asked a question about the concept, he/she tells the concept definition as he/she learned from the teacher but continues to use the concept image he/she has in other cases. (ii) In the second case, unlike in the first case, the student has not encountered the concept before. That is, the concept image cell is empty. In this case, the concept image begins to take shape according to the concept definition.

Apart from the interaction process between concept definition and concept image, there are three different uses of these two during a cognitive activity (Vinner, 1983). In the first, the student uses the concept definition more than his own concept image in the cognitive activity process. In the second, the student focuses only on the concept definition. In the third, the student uses only the concept image. However, in teaching processes, it is stated that the situations where only concept image is used are the most common. Considering this situation, it is seen that concept image has a more active role in the concept teaching process rather than concept definition. In this case, it can be stated that students benefit from concept images more frequently in mathematical operations and learning processes.

It is seen that many different studies have been conducted on Tall and Vinner's (1981) concept definition and concept image model (Ergün, 2010; Fujita, 2012; Fujita & Jones, 2007; Macit & Nacar, 2019; Monaghan, 2000; Nakahara, 1995; Vinner, 1983; Ward, 2004; Wilson, 1990). Among the studies examined, it has been observed that there are studies on different subjects, such as trigonometric functions, rational numbers, and geometric objects, for mathematics education.

It has been taken into account that secondary school mathematics teacher candidates encountered the concept of multivariate function for the first time at the university level. In the literature review, no concept image studies were found for multivariate functions. In addition, it was thought that only one-variable functions were not sufficient in the formation of pre-service mathematics teachers' concept images about function. Therefore, in this study, both univariate functions and multivariate functions are focused because learning multivariate functions is associated with univariate functions (Yerushalmy, 1997). To learn multivariable functions, besides basic function knowledge, the properties of univariate functions should be adapted to multivariate functions. Thus, learning multivariate functions can be expressed as challenging for most students. In addition, spatial thinking and visualization skills are required for the transition from univariate functions to multivariate functions. Therefore, geometric interpretation of multivariable functions may not be quick and easy for some of the students. The analysis course, one of the mathematics education courses at the university level, has a structure based on the subject of functions. The subject of univariate functions and multivariate functions is also included in different departments of universities related to mathematics. Most of the students studying in these departments are faced with multivariate functions for the first time. In general, multivariable functions are also the basis for many different areas of mathematics. Hence, the challenges that may be encountered in the learning process of multivariate functions will negatively affect the learning of other concepts related to this concept. This study aims to examine the pre-service teachers' concept images regarding the concepts of univariate function and multivariate function at the university level. According to the findings and results obtained from this study, suggestions will be presented to facilitate the transition from univariate functions to multivariate functions. And also, examining the concept images of the students about the single and multivariate functions can be useful in the planning of the teaching content and activities.

According to these purposes, the sub-problems of the research are as follows:
1- What are the concept images of secondary school mathematics teacher candidates regarding the concept of function?
2- What are the concept images of the secondary school mathematics teacher candidates on the example of a single variable function?
3- What are the concept images of the secondary school mathematics teacher candidates on the example of a multivariate function?

2. Method

2.1 Research Design

Qualitative research method, which aims to determine the concept images of pre-service mathematics teachers on single and multivariate functions, and a descriptive survey model were used in this research. The main purpose of the descriptive survey model is to accurately observe and describe a situation and its characteristics. In the field of education, it is used to learn the affective and cognitive characteristics of people (Johnson & Christensen, 2014). In this study, the descriptive survey model was used as the concept images of pre-service teachers about single and multivariate functions were examined. Concept images of pre-service secondary school mathematics teachers were expressed with descriptive statistics.

2.2 Participant (Subject) Characteristics

The study group of this research consisted of pre-service secondary school mathematics teachers. Pre-service teachers participated in the present study voluntarily. Easily accessible case sampling, one of the purposeful sampling methods, was used. This sampling method adds speed and practicality to the research. In this method, the researcher chooses a situation that is close and easy to access (Yıldırım & Şimşek, 2011). This study was conducted with 37 secondary school mathematics teacher candidates. All of the candidates had taken the analysis 1 course, which included the concept of function. In the selection of the sample, no elimination was made from the pre-service teachers in the class and all the pre-service teachers in the class were included in the scope of this study, but the pre-service teachers who were not present when the test was applied were not included in the scope of this study.

2.3 Data Collection Tools

The data of the study were obtained with a test consisting of open-ended items. A test consisting of three open-ended questions was prepared by the researcher to obtain data on the definition of the pre-service teachers' concept of a function and their concept images for single and multivariate function examples. In this test, firstly, the definition of function concept, secondly, univariate function concept and its example, and thirdly, multivariate function concept and its example were asked. For the content validity of this test, the relevant literature and the opinion of an expert in mathematics education were deemed sufficient. In applying the test, first of all, pre-service teachers were informed about this study, and they were explained to write concept definitions and explain the concept examples in detail. Then, to determine the concept images of the pre-service teachers about their functions and sub-concepts, a test consisting of three open-ended questions was applied to them in 30 minutes after the lesson.

2.4 Data Analysis

The data obtained from the test consisting of open-ended questions were examined in three stages: understanding, planning and evaluation. The concept images of the pre-service teachers were examined at each stage.

As a result of the literature review and reading of the data by the researcher, a coding for the data was created. Each word, sentence and examples were coded accordingly. Then thematic coding was performed. The codes related to each other were brought together to reach the themes. The study performed at this stage is the content analysis study. Content analysis is the classification of verbal and written data in terms of a specific problem or purpose and categorizing them to extract a specific meaning from them (Tavşancıl & Aslan, 2001). The data obtained from the test were coded and analyzed qualitatively. The coding process was carried out together with the researcher and a mathematics education specialist. The data obtained from the present study were coded by two researchers working independently from each other. The number of “consensus” and “disagreement” situations was determined for the coding made. According to Miles and Huberman (1994) coding reliability formula, it was calculated as 96%. Each coding is presented in themes with frequency and percentage values. Concept images and examples of pre-service teachers' functions and sub-concepts were analyzed and interpreted descriptively.
The main purpose of this research is to present a descriptive and realistic picture of the researched subject. Since the purpose of generalization is not carried out, the findings are limited only to the pre-service teachers who participated in this research. In qualitative research, the validity and reliability of the findings are ensured by presenting the data obtained as detailed and directly as possible (Yıldırım & Şimşek, 1999). Given this situation, quotations from the answers of the pre-service teachers were also included.

2.5 Statements of Publication Ethics

This research was reviewed by the Izmir Demokrasi University Social and Humanities Ethics Committee, and it was decided that the research was ethically appropriate. Date and ethical approval decision number for this study: 08/04/2022- 2022/04-04

3. Results

In this study, in which the pre-service teachers' concept images for univariate and multivariate functions were examined, first of all, the pre-service teachers' concept images for the function were examined. The data obtained are presented in Table 1.

Table 1: Function Concept Images of Pre-service Teachers

Themes for the Concept of Function	f	%
Input-output (machine, conversion)	14	37.8
Matching	13	35.1
Equation	4	10.8

A type of transaction	1	2.7
Graphic representation	1	2.7
Rule	1	2.7
Blank response	3	8.1

When Table 1 is examined, it is seen that the pre-service teachers' concept images about the function consist of input-output (machine, transformation), matching, equation, a type of operation, graphical representation and rule expressions, respectively. The concept images of the pre-service teachers were mostly gathered under the themes of input-output and matching, and three pre-service teachers did not answer this question. Although the pre-service teachers do not include the details of the domain and image set of the functions, it can be said that they mostly gave the matching and input-output answers based on these contents. Only one pre-service teacher preferred to express it according to the way of representation by drawing graphics instead of defining the function.

The concept images of the pre-service teachers regarding the univariate function examples are examined in Table 2.

Table 2: Concept images of pre-service teacher for the example of a univariate function

Themes for the Univariate Function Example	Examples of functions with one variable	f	%
Examples of equations based on y and f(x) notation	f(x)=2x+3 y=2x+12 y=x+9 $4t^2$ + 3t-2=y (a different variable instead of x) f(x)=x+3 f(x)= 2x+x^2+3 (quadratic function) f(x)=7x+3 f(x)= 5x+6 f(x)= 2x+2 f(x) =x+5 f(x)=2x+9 f(x)=3x+5 f(x)= x+8 f(x)=7x+5 f(x)=2x+7	15	40.5
Examples of algebraic expressions	ax+b, x^2 -12x 5x+7 ax 2x 5x+2 3x+5 4x+5 x+5 3x	10	27.02
Examples of equations	3x+2=6 2x=5 x+3=5 2x+1=3 3x+2=5	8	21.6

	ax+b=c 3x+5=8 x=1		
Multivariate expression	2x+4y=6	1	2.7
Verbal expression	A function is to show results by sticking to a single value in a graph.	1	2.7
Blank response		2	5.4

When Table 2 was examined, it was seen that pre-service teachers mostly gave equality examples based on f(x) and y notation as univariate function examples. Ten pre-service teachers wrote algebraic expressions as examples of univariate functions. In this case, it can be thought that giving an example of a function with an algebraic expression without depending on f(x) is a result of incomplete or wrong learning. While expressing the concept of univariate function, eight pre-service teachers used equation examples. One pre-service teacher gave an example of a multivariate function, one pre-service teacher preferred verbal expression, and two pre-service teachers did not respond. When the examples given in general were examined, it was seen that while the majority of the pre-service teachers used the letter x as variable/unknown, only one pre-service teacher used the letter t. In addition, most of the examples given were linear functions/equations, and only one pre-service teacher gave an example of a quadratic function. While it is possible to express the concept of function with different representations, such as graphs, set diagrams, tables and words, most pre-service teachers used algebraic representation in sample selection.

Concept images of pre-service teachers regarding multivariate function examples are examined in Table 3.

Table 3: Concept images of pre-service teachers for examples of multivariate functions

Themes for Multivariable Function Example	Examples of Multivariable Functions	f	%
Examples of equations based on the y or g(x) notation	$4t^2+ 3k=y$ g(x)=5x+4y+2 $g(x)=4x^5+3x^4+7x^3+11x+20$ 2x+12=y x+5=y $y=x^2+3$ g(x)=x+y+3 g(x)= 2x+3y+z f(x)= 4a+3b $fgx)= 3x^2+2x+4$ g(x)= 2x+3y+xy $g(x)=x^2+4x-4$ f(x)=2x+3y+5 $h(x)=3x^2+5y$ f(x)= 2x+3y+4	15	40.5
Examples of algebraic expressions	ax+ by +c y^2-6x 8x+5y+10 10x-7y+9 3x+4y $2x^2+4y+1$ 2x+y 3x+5y 2x+3y	9	27.02

Examples of equations	2x+3y=8 x.y=9 x+2y=13 2x+5y=47 x^2+1=2 (x=+-1) 2x+4x=0 x^2+y3=27 4x-5-3y=0 ax+by+cz=0	9	27.02
Multivariable function	f(x,y)= 2x+3y+8	1	2.7
Verbal expression	Displaying different kinds of values in a chart	1	2.7
Blank response		2	5.4

When the pre-service teachers examined the concept images of the multivariate function examples in Table 3, it was observed that some basic concepts about the function were not sufficiently known. For example, some pre-service teachers defined the variable notation of the multivariate function as x only, as in the example g(x)=5x+4y+2. Afterwards, they expressed the multivariate function with f(x), g(x) or h(x), but in the continuation of the equation, they wrote two different variables (expressions containing x and y). This shows that they have misconceptions not only about the multivariate function but also about the way the function is expressed. Another missing information that pre-service teachers have about the function is the definition and value sets of the function because pre-service teachers did not specify definitions and value sets for both univariate and multivariate functions. Although the pre-service teachers generally used the variables x and y in the examples of multivariate functions, they used incorrect expressions as a form of representation. Although he did not specify the definition and value set, only one student gave a correct example with the notation f(x,y)= 2x+3y+8. In that case, it is possible to say that pre-service teachers have incomplete knowledge not only about multivariate functions but also about the concept of function in general. The findings suggest that teaching the subject of multivariate functions directly, without completing these deficiencies about the basic meaning of the function, can continue the incomplete information and misconceptions.

4. Discussion

The data obtained in this research, in which the concept images of the pre-service teachers for the concept of a function and the examples of single and multivariate functions were examined, it was determined that first of all, the concept images of the pre-service teachers for the function consisted of different features/aspects of the concept of function. Function concept images of pre-service teachers were generally under the themes of matching, transformation and equation. Carlson, Oehrtman and Thompson (2005) stated that students should be supported to distinguish between function and equation, regarding the mistake of perceiving function as an equation. As another finding in this study, a small number of pre-service teachers had the concept images of rules and procedures for the concept of function. According to these findings obtained from the first question of this research, the majority of pre-service teachers had incomplete/wrong learning in the basic definition of the concept of function. In addition, the concept images of some of the pre-service teachers matched the concept definition of the function. However, this image overlapped with only one of the multiple representations of the function and this image did not have such versatile content as the concept definition of the function. While it is possible to express the concept of function under various representations, such as graphics, cluster diagrams, tables, and verbal and algebraic representations, the majority of pre-service teachers used algebraic representation in function examples. According to the National Council of Teachers of Mathematics [NCTM] (1989) standards, students are expected to have the ability to represent functions with tables, graphs, equations and verbal expressions, to know the meanings of these representations and to make transitions between representations. In addition, Lobato and Bowers (2000) stated that it is important to use multiple representations and although students know how to transform between representations, students may not be able to explain and realize what, how and how they are represented.

In this regard, Christou, Elia and Gagatsis (2004) emphasize the necessity of alternative teaching approaches in teaching the use of multiple representations and transformation between representations.

The results obtained from the current research on the concept of function were also similar to other studies in the literature (Sierpinska, 1992; Vinner & Dreyfus, 1989). Also, in a similar study, it was stated that "Mistakes regarding the formal definition of the concept of function arise from students' describing the concept incompletely or incorrectly. For example, the function is only a match, a formula or an equation" (Kabael, 2010, p.). This view supported the results of the present study. Similarly, the inference in Mayes's (2001) study that students defined the function only as a rule or a formula was similar to the present study. In a different study on function concept images (Aydın & Köğce, 2008), the findings showed that most of the pre-service teachers perceived functions as equations and that they were inadequate in defining function concepts. The findings of that study also overlapped with the results of the current research. In this case, it is possible to say that some concept images of the pre-service teachers regarding the concept of function take the meaning of the function in one direction and make the definition limited.

For the second sub-problem of the research, concept images of pre-service teachers for univariate functions were investigated. In their answers, pre-service teachers did not write anything about definition and value sets, which are important components of the concept of function; they wrote equations based on f(x) or y notation. It has been determined that some of the pre-service teachers have an equation concept image related to the concept of function. Another result obtained from this research is that some pre-service teachers have the concept image of algebraic expressions related to the concept of function. According to these results, it is possible to say that the pre-service teachers have deficiencies and misconceptions in their basic knowledge of the concept of function, which is one of the main topics of the analysis course. The reason for this deficiency can be considered the fact that the function notations are written without conceptual knowledge and emphasizing the conceptual content in the questions, the concept definition is not emphasized enough in the teaching of the function concept, the function is not explained with multiple representations or the transition between representations is not emphasized enough. In short, it can be said that explaining the operational properties of the function more than the conceptual aspect of the function causes information deficiencies.

When the pre-service teachers' preferences for the letter and the degree of function in the univariate function examples were examined, only one pre-service teacher wrote an example of a quadratic function. Other pre-service teachers wrote linear functions to univariate function examples. In addition, it has been determined that the pre-service teachers' letter preferences are generally the letter x as a variable, and they mostly choose f(x) notation as a notation. Therefore, it can be said that pre-service teachers' concept images are affected by different expressions, such as a variable letter, representation style and degree of function, which are more commonly used in questions. In other words, it can be stated that pre-service teachers' concept images are formed according to the common usage of a concept in books, questions or course content.

As the third sub-problem of this research, pre-service teachers' concept images of multivariate functions were investigated. Pre-service teachers defined function representations in multivariate function examples as f(x) by heart and in a stereotyped way. They used the notation f(x) for multivariate functions, ignoring that x represents the only variable in the function. When the data were examined, it was seen that the pre-service teachers did not use the f(x,y) format, which is correct for multivariate functions, but rather used g(x) and h(x) notations. They preferred different letters instead of the commonly used f(x) notation in multivariable function examples. For univariate functions, most of the pre-service teachers wrote the notation f(x). In the case of multivariable functions, they thought that they were making a correct representation only by letter change, whereas their representation was not correct. In function representations, f, h, g only represent the function symbolically. The most important thing is to write one or more variables correctly in the function representation according to the number of variables in the function. In other words, the expected response from the pre-service teachers for the representation of bivariate functions was not h(x), f(x) or g(x). They were expected to write f(x,y) for bivariate functions. The reason for the letter preferences in the pre-service teachers' examples was that they knew that single and multivariate functions had different representations. However, they did not have the knowledge of notation to express this difference. They wanted to express this difference simply by changing the letter f in the notation f(x). In general terms, the pre-service teachers did not have knowledge of the f(x,y) form of the functions of two variables. In addition, the pre-service teachers who misunderstood only equation and algebraic expressions as function expressions in univariate function examples also made the same mistake in bivariate function examples. Although they used two variables in their example, they could not write a correct notation. The pre-service teachers did not write the definition and value set in the multivariate function examples as well as in the univariate function examples. Pre-service teachers mostly preferred to use quadratic expressions in multivariate function examples, unlike univariate functions. We can say that pre-service teachers' concept images of multivariate functions are close to quadratic expressions. That is, they took into account the word "many" in the expression "multivariate function." But they did not increase the number of variables in the function according to the word multiple variables; they preferred to increase the degree of the function.

The subject of multivariate functions is taught at universities in departments related to mathematics, such as engineering, basic sciences, and mathematics teaching. Most of the students studying in these departments encounter this subject for the first time. In general, the subject of multivariate functions is an important subject for different fields of mathematics (e.g., analysis, complex analysis, functional analysis, geometry and algebra). The subject of univariate functions, which is the basis of multivariate functions, is also an important subject for many subjects and courses related to the science of mathematics. Therefore, pre-service teachers have learning deficiencies and misconceptions about multivariate functions and univariate functions may negatively affect the learning process of other mathematical concepts related to these concepts.

The main concept that this research aimed to examine was multivariate functions. However, while designing this research, it was desired to examine the concept images of the function definition and univariate function examples before examining the pre-service teachers' concept images of multivariate functions because it is aimed to draw a more holistic conclusion according to the information to be obtained and the incomplete learning and misconceptions to be determined. As predicted in the research design, the pre-service teachers' misconceptions and misconceptions in the concept images of the multivariate function stem from incomplete information and mislearning in the function definition and univariate function examples. Based on this information and data, although the concept of function, which is one of the main subjects of mathematics and some of its sub-branches, is crucial, there are incomplete learning and misconceptions about this concept, and learning this concept is difficult for students (Mayes, 2001). The results obtained in this study are similar to the literature given that, in this study, it has been determined that the incorrect concept images of the pre-service teachers in the research stem from previous incomplete or incorrect learning. Overcoming this difficulty seems to be possible with the existence of situations where concept image and concept definition overlap with each other.

Based on the research results, it should always be considered that mathematics is a course with holistic and progressive content. In other words, for a subject to be learned completely and correctly, there should be no learning deficiency and misconceptions about the previous subjects. In this respect, it can be suggested that lesson plans be made in a way that complements the missing learning and supports previous learning. To teach a concept in the literature, it is recommended to make many examples of that concept (Tall & Vinner, 1981). This recommendation should also be taken into account in this study. The findings suggest that teaching with a large number of examples of single and multivariate functions would be beneficial in terms of an effective learning process. In addition, as a result of this research, it was found that the letter preferences of the pre-service teachers in the function representations were generally the letter x and they mostly chose the f(x) notation as the notation. In other words, the concept images of the pre-service teachers were formed according to the factors, such as letters, representation style and degree of function frequently used in the questions and various sources. In short, the common use and representations of the concept were effective in the concept images of the learners. In questions where the operational feature of the function concept comes to the fore, giving additional information about the concept definition of the function and emphasizing its definition enables learners to learn the concept of function more effectively. In this way, the concept images of the learners can become compatible with the concept definition. Thus, in questions that require procedural knowledge, conceptual information can be included as additional information or as a reminder, provided that there is no clue in solving the problem. This situation can contribute to the concept of images in individual’s mind. Therefore, while preparing some questions, it may be suggested to include conceptual definitions in the question. In this way, the definitions that learners frequently encounter are useful for their concept images. In addition to this research result, it can be suggested that different studies should be conducted in larger research groups on concept image and conceptual and procedural knowledge.

This situation, together with the results of the current study, will contribute to the learning processes in terms of providing a holistic approach to the subject of functions.

Conflicts of interest

The author has not declared any conflicts of interests.

References

Aydın, M. & Köğce, D. (2008). The perceptions of teacher candidates about "equation and function" concepts. *Van Yuzuncu Yil University Journal of Education,* 5, 46-58

Carlson, M., Oehrtman, M., & Thompson, P. W. (2005). Key aspects of knowing and learning the concept of function. *Mathematical Association of America Research Sampler*, *9*.

Christou, C., Elia, I., & Gagatsis, A. (2004). The nature of multiple representations in developing mathematical relationships. *Quaderni di Ricerca in Didattica*, 14, 150-159.

Ergün, S. (2010). *7th grade students' perception, definition and classification of the polygons* [Master thesis]. Dokuz Eylul University.

Fujita, T. & Jones, K. (2007). Learners' understanding of the definitions and hierarchical classification of quadrilaterals: Towards a theoretical framing, *Research in Mathematics Education*, 9 (1-2), 3-20.

Fujita, T. (2012). Learners' level of understanding of the ınclusion relations of quadrilaterals and prototype phenomen. *The Journal of Mathematical Behavior*, 31, 60- 72

Johnson, B., & Christensen, L. (2014). *Educational research: quantitative, qualitative and mixed approaches*. SAGE Publications.

Kabael, T. U. (2010). Concept of the functıon: hıstorıcal development, understandıng process, mısconceptıon and the teachıng strategıes, *Tübav Journal of Science*, 3 (1), 128-136.

Lobato, J., & Bowers, J. (2000, April). Three Perspectives in Research on Functions: Multi-Representational, Quantitative, and Phenomenological. *Annual meeting of the American educational research association*, New Orleans, LA.

Macit, E., & Nacar, S. (2019). Concept images for rational number and fraction of the students at the elementary mathematics education department. *Inonu University Journal of the Graduate School of Education*, *6*(11), 50-62.

Mayes, R. (2001). Using Information technology in mathematics education. (ed: D. James Tooke and Norma Enderson (Eds.), *CAS Applied in a Functional Perspective College Algebra Curriculum*, The Haworth Press, In

Miles, M., & Huberman, A. M. (1994). *Qualitative data analysis: an expanded sourcebook*. Thousand Oaks, CA: Sage

Monaghan, F. (2000). What difference does it make? children's views of the differences between some quadrilaterals. *Educational Studies in Mathematics*, 42(2),179-196.

Nakahara, T. (July, 1995). Children's construction process of the concepts of basic quadrilaterals in Japan. *Proceedings of the 19th Conference of the International Group for the Psychology of Mathematics Education* (Vol:3). Brazil: Universidade Federal de Pernanbuco

National Council of Teachers of Mathematics (NCTM) (1989*). Curriculum and Evaluation Standards for School Mathematics*, Reston, Virginia: NCTM.

Rösken, B., & Rolka, K. (2007). Integrating intuition: The role of concept image and concept definition for students' learning of integral calculus. *The Montana Mathematics Enthusiast*, *3*, 181-204.

Sierpinska, A. (1992). On understanding the notion of function. *Aspects of Epistemology and Pedagogy*, 25, 23-58.

Tall, D. & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, 12(2), 151-169

Tavşancıl, E. ve Aslan, E.A. (2001). *Content analysis and application examples for verbal, written and other materials.* Epsilon Publishing

Vinner, S. & Dreyfus, T. (1989). Images and definitions for the concept of function. *Journal for Research in Mathematics Education*. 20(4), 356-366.

Vinner, S. (1983). Concept definition, concept image and the notion of function. *Internetional Journal of Matematical Education in Science and Technology*, 14(3), 293-305.

Vinner, S. (2002). The role of definitions in the teaching and learning of mathematics. *In Advanced Mathematical Thinking* (pp. 65-81).

Ward, R. A. (2004). An investigation of K-8 preservice teachers' concept images and mathematical definitions of polygons. *Issues in Teacher Education*, 13(2), 39-56.

Wilson, P.S. (1990). Inconsistent ideas related to definitions and examples. *Focus on Learning Problems in Mathematics,* 12(3), 31-47

Yerushalmy, M. (1997). Designing representations: reasoning about functions of two-variables. *Journal for Research in Mathematics Education*, 28, 431-466.

Şimşek, H., & Yıldırım, A. (2011). Qualitative research methods in the social sciences. *Seçkin Pub*.