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Analysis of Turkish Middle School Mathematics Textbooks in Terms of Opportunities to Learn "Area Measurement"

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Abstract
This study aims to examine the 5th- and 6th-grade mathematics textbooks approved by the Turkish Ministry of National Education for the 2021–2022 academic year in terms of the opportunities to learn provided in the sub-learning domain of "Area Measurement." A document analysis was performed as a part of this study. Consistent with the aim of the study, educational content (instructions, worked examples, exercises, activities, and unit evaluation) related to the area measurement sub-learning domain in five different 5th- and 6th-grade mathematics textbooks were examined in terms of the learning opportunities they offered. Dedoose, a web-based program, was used in the data collection phase of the study. The data analysis revealed that in the examined textbooks, the tasks on the topic of area measurement have an emphasis on procedural knowledge. It was determined that the activities and worked examples in the examined textbooks on the said topic had only one outcome and one solution method. In terms of contextual features, textbook tasks on the topic of area measurement were found to be mostly intra-mathematical. In light of the findings, certain recommendations were made.

Keywords: Area Measurement, Math Textbooks Opportunities to Learn

1. Introduction

Measurement is the process of assigning a numerical value to a continuous quantity (Van De Walle et al., 2014). One of the fundamental topics of mathematics education, measurement is taught to students starting from the first years of school and continues until middle (5^{th}-8^{th} Grades) school. Measurement has a wide range of applications in various mathematical subjects as well as in daily life situations and problems (Lehrer, 2003). Area measurement is frequently used particularly in daily life and utilized at all levels of mathematics from basic to advanced. Therefore, the importance of teaching the topics of area and area measurement in all grades is undeniable (Smith et al., 2016). When the Turkish curriculum for middle school mathematics is examined, the learning outcomes of area and area measurement are addressed within the context of the area measurement sub-learning domain of Geometry and Measurement strand, in 5^{th}- and 6^{th}-grade levels (Ministry of National Education [MoNE], 2018). In addition to the mentioned area measurement sub-learning domain, target learning outcomes related to area measurement in other grade levels as well. Within the context of the area measurement sub-learning domain, in the 5th grade, students are taught how to calculate the area of rectangles, whereas, in the 6th grade, they learn about measuring the areas of triangles, parallelograms, and lands.

Findings of some studies on the instruction of the concept of area show that students have several misconceptions on the concept of area and area measurement and that they have challenges in solving relevant problems (Chappell & Thompson, 1999; Hirstein et al., 1978; Lin & Tsai, 2003; Moreira & Contente, 1997; Moyer, 2001). One of the reasons why students have difficulty in learning area measurement is that teachers mostly use procedural methods instead of conceptual ideas when teaching this topic (Hong et al., 2018; Murphy, 2012). The failure of primary school students in area measurement is known to be because of the rote learning practices used in teaching geometry formulas (Barrantes & Blanco, 2006). Mere recall of the formula to calculate the area of a rectangle, that is, finding the length and width and then multiplying them together is an example of rote teaching practices. With such an approach, students experience difficulty when making sense of why they have to multiply the length of the rectangle by its width to calculate the area (Stephan & Clements, 2003). The basic concepts of area measurement are related to the number of units of equal size needed to cover an area without gaps or overlaps, dividing a region into units of equal size, counting unit squares, understanding repeating units, rows, and columns, and associating the number of unit squares with length and width (Battista, 2007; Sarama & Clements, 2009). By teaching area measurement at the appropriate sequence and by particularly drawing on these concepts, student misconceptions can be prevented. However, some Turkish studies revealed that students were trying to reach the outcome through rote methods, without making sense of the formulas used for area measurement. Therefore, area measurement is among the topics that students have difficulty making sense of and wherein students make the most mistakes (Dağlı, 2010; Tan-Şişman & Aksu, 2009).

Another reason for the difficulties in learning area measurement is that the content related to Geometry and Measurement in textbooks is conceptually limited (Hong et al., 2018; Smith et al., 2016). Thus, employing teaching methods that will eliminate students' misconceptions about area measurement and utilizing various supporting materials (e.g., textbook and dynamic mathematics software) in teaching the topic are crucial. In mathematics courses, textbooks, which are crucial in determining and organizing the course content (e.g., activities and questions.), also serve as the core instructional materials used by teachers (Keitel et al., 1980; Valverde et al., 2002). Textbooks that are considered a potential tool in achieving the learning outcomes formulated when designing course content must have effective educational content (e.g., instructions, questions, activities, and solved examples) (Bergwall, 2019; Haggarty & Pepin, 2002; Kilpatrick et al., 2001; Newton & Newton, 2007; Peterson et al., 2020; Son & Diletti, 2017; Vicente et al., 2019). Playing a key role in teaching, textbooks can contribute to designing high-quality learning activities only if their content is consistent with the intended learning outcomes and principles (Swan, 2007; Valverde et al., 2002). Therefore, an important approach of identifying students' opportunities to learn mathematics is by examining textbooks. The learning opportunities provided to students by textbooks, which are crucial to teaching, can be identified (Hong et al., 2018; Remillard & Heck, 2014). If textbooks limit students' ability to understand various mathematics topics, it may negatively affect their learning outcomes and cause them to experience difficulties in learning these topics (Smith et al., 2013). Considering the importance of textbooks in designing learning activities and identifying the target learning outcomes, examining the opportunities to learn offered by the mathematics textbooks on a given topic is crucial. Accordingly, this study aims to examine the 5th- and 6th-grade mathematics textbooks approved by the Turkish Ministry of National Education for the 2021–2022 academic year in terms of the learning opportunities they provide on the sub-learning domain of "Area Measurement." The study sought to answer the following questions:

1) What is the place of the area measurement sub-learning domain in the middle school mathematics curriculum?

2) What are the order and timing of the tasks in mathematics textbooks related to the sub-learning domain of area measurement?

3) What is the distribution of tasks related to the sub-learning domain of area measurement in mathematics textbooks by types of knowledge?

4) What is the distribution of tasks related to the sub-learning domain of area measurement in mathematics textbooks by contextual features?

5) What is the distribution of tasks related to the sub-learning domain of area measurement in mathematics textbooks by the number of solution methods and outcomes?

2. Method

2.1. Research Model

This study aims to examine the 5th- and 6th-grade mathematics textbooks approved by the Turkish Ministry of National Education for the 2021–2022 academic year in terms of the learning opportunities they provide on the sub-learning domain of "Area Measurement," the document analysis, a qualitative research method, was used. The document analysis refers to the analysis of written materials containing information about investigated case or cases (Yıldırım & Şimşek, 2018). As in other qualitative research methods, the document analysis requires examining and interpreting data to reveal the meaning and obtain empirical knowledge (Corbin & Strauss, 2008; Rapley, 2007).

2.2. Data Sources

Consistent with the aim of the study, a total of five mathematics textbooks, two of which are for the 5th grade and three for the 6th grade, were examined. Table 1 presents the information regarding the textbooks examined as a part of the study and the codes assigned to the said textbooks.

Table 1: Textbooks Examined in the Study

Textbook	**Code**
Cırıtcı, H., Gönen, İ., Araç, D., Özarslan, M., Pekcan, N., & Şahin, M. (2021). Ortaokul ve imam hatip ortaokulu 5.sınıf matematik ders kitabı [*Middle and imam hatip secondary school 5th grade mathematics textbook*]. Ankara: MEB yayınları.	5G1
Erenkuş, M. A., & Savaşkan, D.E. (2018). Ortaokul ve imam hatip ortaokulu 5.sınıf matematik ders kitabı [*Middle and imam hatip secondary school 5th grade mathematics textbook*]. Ankara: Koza Yayınları	5G2
Çağlayan, N., Dağıstan, A., & Korkmaz, B. (2021). Ortaokul ve imam hatip ortaokulu 6.sınıf matematik ders kitabı [*Middle and imam hatip secondary school 6th grade mathematics textbook*]. Ankara: MEB yayınları.	6G1
Bektaş,M., Kahraman, S., & Temel, Y. (2021). Ortaokul ve imam hatip ortaokulu 6.sınıf matematik ders kitabı [*Middle and imam hatip secondary school 6th grade mathematics textbook*]. Ankara: MEB yayınları.	6G2
Şahin, M., & Doğan, S. (2019). Ortaokul ve imam hatip ortaokulu 6.sınıf matematik ders kitabı [*Middle and imam hatip secondary school 6th grade mathematics textbook*]. Ankara: Engürü Yayınları	6G3

(5G1: 5thGrade 1, 5G2:5th Grade 2, 6G1:6th Grade 1, 6G2: 6th Grade 2, 6G3: 6th Grade 3)

2.3. Framework for Data Analysis

In the textbooks, sections with similar content were given under several different titles ("Information box;" "Note;" "Let's remember;" "Mine of information;" "For those who wonder;" "Let's do it together;" "Let's learn together;" "Now it's your turn;" "Exercises;" "Let's put to use;" "Topic evaluation;" "Let's try out;" "Activity;" "Build, live, learn;" and "Unit evaluation"). Therefore, these sections were classified by their content under five different categories, namely "Instructions," "Worked Examples," "Exercises," "Activities," and "Unit Evaluation," and examined accordingly. Figure 1 presents this classification.

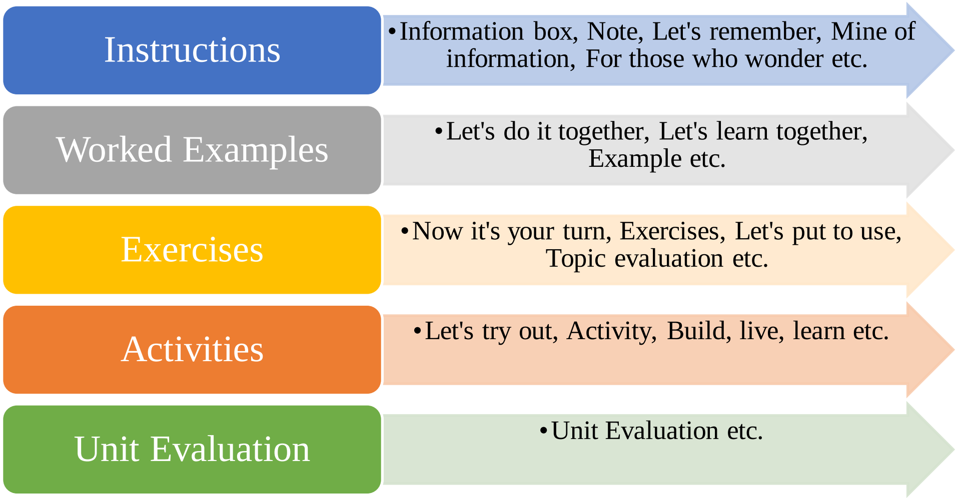


Figure 1: Classification of the Contents in the Textbook

For the analysis of the data, a framework developed based on the findings of studies conducted by Hong et al. (2018), Glasnovic-Gracin (2018), Bingölbali (2020), and Smith et al. (2016) was used Table 2 presents the sub-dimensions of the said framework.

Table 2: Framework Used in Textbooks to Analyze Learning Opportunities

Analysis framework of area measurement contents
Learning outcomes/number of lessons related to area measurement
Timing and topic sequence
Types of knowledge (conceptual, procedural)
Response types (a single correct outcome, multiple correct outcomes, a single solution method, multiple solution methods)
Contextual features (intra-mathematical, realistic, authentic)

This framework is based on the framework used by Hong et al. (2018) to compare textbooks in terms of learning opportunities related to area measurement. However, the information types, response types and contextual features dimensions of the framework have been reshaped. The types of the knowledge dimension of the framework were based on the study conducted by Smith et al. (2016), wherein they examined conceptual and procedural knowledge in detail. The number of solution methods and outcome dimension was based on Bingölbali (2020), and the contextual features dimension was based on Glasnovic-Gracin (2018).

2.3.1. Timing and Topic Sequence

For students in the 4th, 5th, and 6th grades, the primary learning outcome in terms of the area measurement sub-learning domain is developing and using formulas to calculate area (MoNE, 2018; National Council of Teachers of Mathematics [NCTM], 2000). In Turkey the concept of area was first introduced in the 3rd grade; the target learning outcomes are determining the area of two geometric shapes of the same type and different sizes by calculating the number of unit squares required to cover them, estimating area using nonstandard units of measurement, and checking the correctness of the estimation. In the 4th grade, the relevant learning outcomes are determining that the area comprises square units and associating the area of squares and rectangles with multiplication and addition. After achieving these learning outcomes, middle school students are expected to be capable of calculating the area of and the number of unit squares needed to cover planar shapes using appropriate nonstandard units, estimating an area with nonstandard area measurement units, and checking the correctness of the estimation by counting the unit squares, concluding that the areas of the shapes are equal to the number of unit squares occupying that area, and associating the area of squares and rectangles with addition and multiplication operations (MoNE, 2018).

2.3.2. Types of Knowledge

In the previous decade, procedural and conceptual knowledge, which are essential for teaching and learning, have become much more important (Ball et al., 2008; Lin et al., 2013; Shulman, 1987). Conceptual knowledge can be defined in a nutshell as a conceptual understanding that encompasses the relationships, principles, and meanings of mathematical symbols and algorithms or as a "holistic and functional understanding of the mathematical idea" (Kilpatrick et al., 2001; Lin et al., 2013; Rittle-Johnson & Schneider, 2014). In their study, Smith et al. (2016) addressed conceptual and procedural knowledge related to area measurement. There are three subtypes of conceptual knowledge, namely, general properties of quantities and their measures, measurement of area specifically and measurement of area of specific shapes. General properties of quantities and their measures; expressions such as that the area can receive include only a positive and single value, that the area has not changed under changes in the position and orientation of the region, that the two merged areas form another area in total. Principles specific to area measurement are related to aspects, such as the definition of area, the meaning of area measure, and tools to measure area. The principles of area measurement for specific shapes are related to aspects, such as the spatial attributes of the rectangle, the array structure of the rectangle (rows and columns), and the fact that all triangles with the same base length and height have the same area (Smith et al., 2016).

Figures 2 and 3 show an analysis by Hong et al. (2018) of the types of knowledge and relevant examples. Figure 3 is an example of conceptual knowledge in a mathematics textbook used in the USA. This example, which also comprises instructions, is related to the use of conceptual knowledge because it explains in stages the process of covering an entire shape with repeating unit squares to measure area. In the example, first, the concept of unit is introduced, and then, it is emphasized that the area of the shape can be expressed in terms of units needed to cover the shape without gaps. Particularly the part of the example wherein the area is expressed in units needed to cover the shape is indicative of conceptual knowledge.

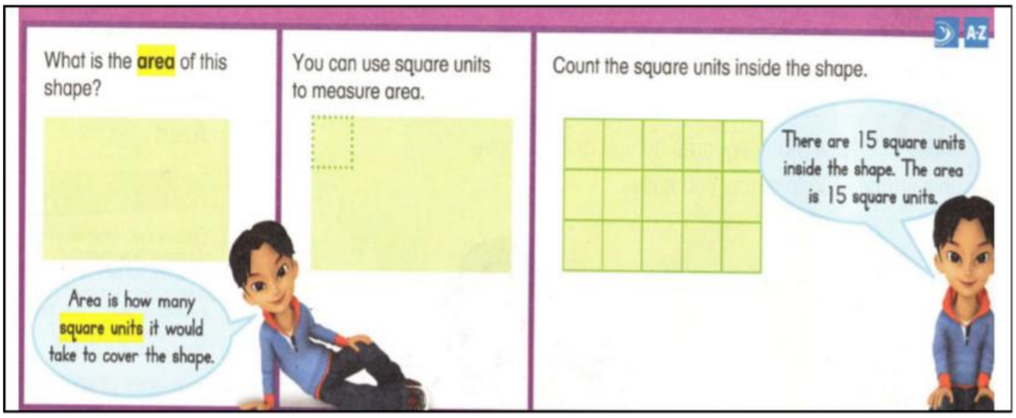


Figure 2: Conceptual Knowledge Example (Hong et al., 2018, s.13)

Pre-measurement, which includes qualitative processes for judging the relative size of two-dimensional objects and shapes, numerical measurement procedures for generating area measures of two-dimensional shapes, and reasoning with area measures that encompasses procedures for generating areas of more complex shapes or reasoning with two or more area measures are indicative of procedural knowledge. Actions such as making visual comparative judgment about two or more regions, determining the total number of unit squares in a row-and-column structure (array) either by counting all the units in it or multiplying the number of units in individual rows and columns, and calculating the area of a region by dividing, moving, and recomposing it are carried out by making use of procedural information (Smith et al., 2016).

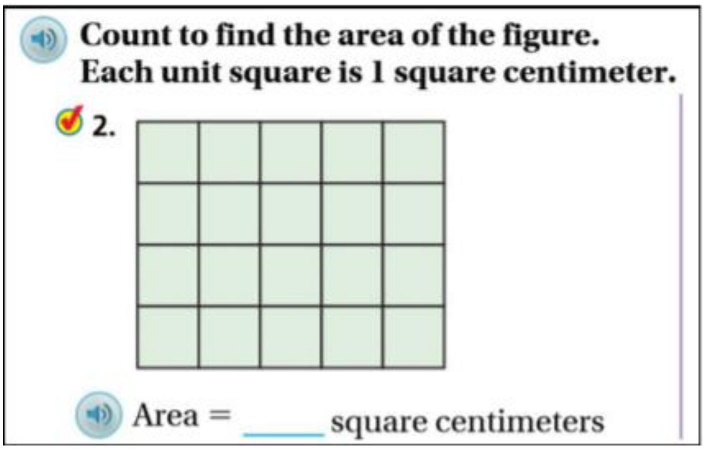


Figure 3: Procedural Knowledge Example (Hong et al., 2018, s.13)

Figure 3 is an example of procedural knowledge in a mathematics textbook used in the USA. This example is related to procedural knowledge because it points to the determination of area by counting square units in a given region. In this example, the aspect of determining area of the region by counting the square units and expressing the area in terms of units points to procedural knowledge.

2.3.4. Response Types

Bingölbali (2020) examined the extent to which middle school mathematics textbooks provide tasks with multiple correct outcomes and multiple solution methods. Accordingly, the tasks in the examined textbooks were analyzed in terms of whether they have multiple correct outcomes, a single correct outcome, multiple solution methods, and a single solution method. Tasks with multiple correct outcomes indicate tasks with at least two correct outcomes and tasks with single correct outcome mean tasks with one correct outcome; whereas tasks with multiple solution methods mean tasks with at least two solution methods, and tasks with single solution methods mean tasks with one solution method.

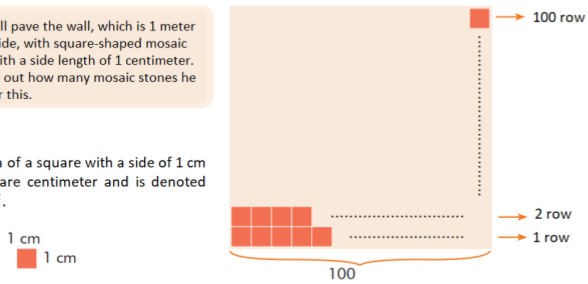
2.3.5. Contextual Features

Contextual features refer to the extent and approaches to which real-world experiences are included in the textbook content. Zhu and Fan (2006) distinguish between application and non-application problems in mathematics. While non-application problems (intra-mathematical) refer to situations that are irrelevant to real-life situations, application problems are related to the context of a real-life situation. Furthermore, the two subtypes of application problems are realistic and authentic application problems; the former refers to data made by the textbook author, whereas the latter refers to data from real-life situations or from students' daily lives.

2.4. Data Analysis

Dedoose software was used for data analysis. Dedoose is a cross-platform application used for the analysis of qualitative and mixed methods research with text, photos, audio, videos, spreadsheet data, and more. For data analysis, the textbook chapters to be examined and the dimensions of the developed framework were uploaded to the Dedoose system along with their assigned codes. All contents of the examined textbooks, except for instructions, were analyzed in terms of the number of solution methods and outcomes and contextual features dimensions of the developed framework. Only 'worked example' and 'activities' tasks were analyzed in terms of whether they had single or multiple solution methods. The instructions in the textbooks were only analyzed in terms of types of information. Table 3 shows examples from the tasks (tasks were translated from Turkish to English and presented) in the textbooks, which were examined within the created framework.

Table 3: Examples of Textbook Content Analyzed According to the Framework

Content	Example	Codes
Instruction	**Information Box** Area of rectangle= (the length of the short side) x (the length of the long side) (5G1, p.291)	Procedural
Activity	**Activity** Tools and equipment • Ruler • Scissors • Broadsheet • Cardboard Application steps • Measure the side lengths of the square and rectangle given below and draw a square and a rectangle with the same dimensions on the broadsheet. • Cut 1 cm^2 square areas from cardboard. • Cover the inside of the square and rectangle you drew on your broadsheet, with the unit squares you cut, so that there are no gaps. • Tell how many square centimeters are the areas of the two shapes. • Say the product of the numbers of the 1 cm^2 regions on both sides of the square. • Find the number of regions of 1 cm^2 on the two sides of the rectangle that meet at one corner. Say the product of these numbers. • Explain how to find the areas of square and rectangle. (5G2, p.240)	Procedural Intra-mathematical Multiple solution methods A single correct outcome
Worked example	**Let's learn together** Hasan will pave the wall, which is 1 meter on one side, with square-shaped mosaic stones with a side length of 1 centimeter. Let's find out how many mosaic stones he needs for this. The area of a square with a side of 1 cm is 1 square centimeter and is denoted as 1 cm^2.  1 meter is equal to 100 centimeters. When 100 of the mosaic stones, each of which is 1 centimeter, are lined up side by side, they form 1 meter, which is one side of the wall to be covered. Each side of the square wall will be covered with 100 stones of 1 cm^2 area. In this case, the total number of stones is 100 · 100 = 10 000 pieces. Considering the area of the wall, Hasan can create 1 m^2 area if he combines 10 000 pieces of 1 cm^2 stones. (6G1, p.182)	Conceptual Realistic A single solution method A single correct outcome
Exercise	**Research-Think** Computer screen resolution is generally preferred as 800 x 600 pixels for small monitors, while this resolution is preferred as 1024 x 768 pixels for large monitors. What might these resolutions mean? Please search. (5G1, p.293)	Conceptual Authentic A single correct outcome
Unit Evaluation	17. Afforestation work was carried out in a rectangular area with 400 m and 300 m side lengths in a 300-decare plot. **What is the area of the part of this land that has not been planted?** A) 180 B) 210 C) 240 D) 270 (6G3, p.206)	Procedural Realistic A single correct outcome

In the “instruction” task example given in Table 3, there is an expression for calculating the area of a rectangle directly with a formula. This task points to the use of procedural knowledge because it guides students to use a formula, that is, the multiplication of width to the length to calculate the area of the rectangle. The “activity” task further provided instructions on how to calculate the area of rectangles and squares by determining the number of unit squares needed to cover them. Thus, it is a procedural knowledge task aimed at generating a formula to calculate the area of rectangles and squares. Because this task does not address a real-life situation, it is intra-mathematical in terms of its contextual feature. Because the task comprises instructions on determining area by covering the shapes with unit squares and using a formula, it has multiple solution methods. Because the instructions of both solution methods lead to one correct answer, the activity has one outcome. In the analyzed "worked example" task, the conversion between the types of units are emphasized. This example, which focuses on finding the number of square units needed to cover the area instead of using quantitative operations involving units of measurement, indicates the use of conceptual knowledge. Because the problem in the task is related to a real-life situation and data was given by the textbook, it is a realistic application problem. Because the task could be solved in only one way and had only one correct answer, it was found to have one solution method and one outcome. The analyzed "exercise" task draws on the definition of area and the size of a given area; therefore, it necessitates the use of conceptual knowledge. Because this task is related to a real-life situation and data was drawn from the real life, it is an authentic application problem. Further, because the task has only one correct answer, it has one outcome. The analyzed "unit evaluation" task is one that necessitates conducting mathematical operations and measurement units conversion. This task, wherein students are required to use the formula to determine the area of the rectangle, is procedural. This example includes a realistic application problem, that is, a problem on the real-life data. Because there is only one correct answer to the question, the task has only one outcome.

2.5. Validity and Reliability

Lincoln and Guba (1985) used the terms credibility, transferability, dependability and confirmability to elaborate on validity and reliability. The concept of credibility, which is presented as an alternative concept to the concept of internal validity, considering that it may be suitable for the nature of qualitative research, is a concept related to the robustness of the reasoning that leads to the conclusion and the quality of the data (Lincoln & Guba, 1985). In order to ensure credibility in the study, the researcher had a long-term interaction with the documents that are the data source and an expert review was applied to the study. The concept of consistency, which is used for internal reliability in qualitative research, refers to the state of being able to consistently give the same result if the study is repeated in line with the regularity of the researcher's interpretations (Batdı & Oral, 2020). In order to ensure consistency in the research, coding was repeated by two different researchers after a certain period of time. With the formula proposed by Miles and Huberman (1994), the percentage of intercoder reliability was calculated for two different researchers and the percentages were determined as 80% and 82%.

3. Findings

In this section, the findings were obtained by examining of the 5th- and 6th-grade mathematics textbooks, which were approved by the Turkish Ministry of National Education for the 2021–2022 academic year in terms of the learning opportunities provided on the sub-learning domain of "Area Measurement" by using the framework developed based on the literature.

3.1. Timing and Topic Sequence

The order and timing of the tasks on the sub-learning domain of area measurement in the examined textbooks are given in Table 4.

Table 4: Timing and Subject Order of 5th and 6th Grade Mathematics Textbooks Related to Area Measurement

Textbook	Timing and topic sequence
5G1	Area of Rectangle Estimating the Area Creating Different Rectangles with the Same Area
5G2	Area of Rectangle Estimating the Area Different Rectangles with Same Area

	Height and Area of Parallelogram
	Height and Area in Triangle
6G1	Area Measurement Units
	Land Measurement Units
	Area Problems
	Area of Triangle
	Area of a Parallelogram
6G2	Area Measurement Units
	Land Measurement Units
	Relationship Between Land Measurement Units and Area Measurement Units
	Area Measurement Problems
	Area of Triangle
6G3	Area of a Parallelogram
	Area and Land Measurement Units

On the area measurement sub-learning domain, 5th-grade tasks include calculating the area of a rectangle in square centimeters and square meters, estimating area, and creating different rectangles with the same area; while 6th-grade tasks are related to calculating the areas of parallelograms and triangles, as well as units of area and land measurement, and area problems. As is seen in Table 4, tasks in the examined mathematics textbooks were found to be compatible with the identified learning outcomes.

3.2. Distribution of Tasks in Mathematics Textbooks by Types of Knowledge

The distribution of tasks in the five textbooks examined as a part of this study by types of knowledge, that is, whether they necessitate conceptual or procedural information, is given in Figure 4.

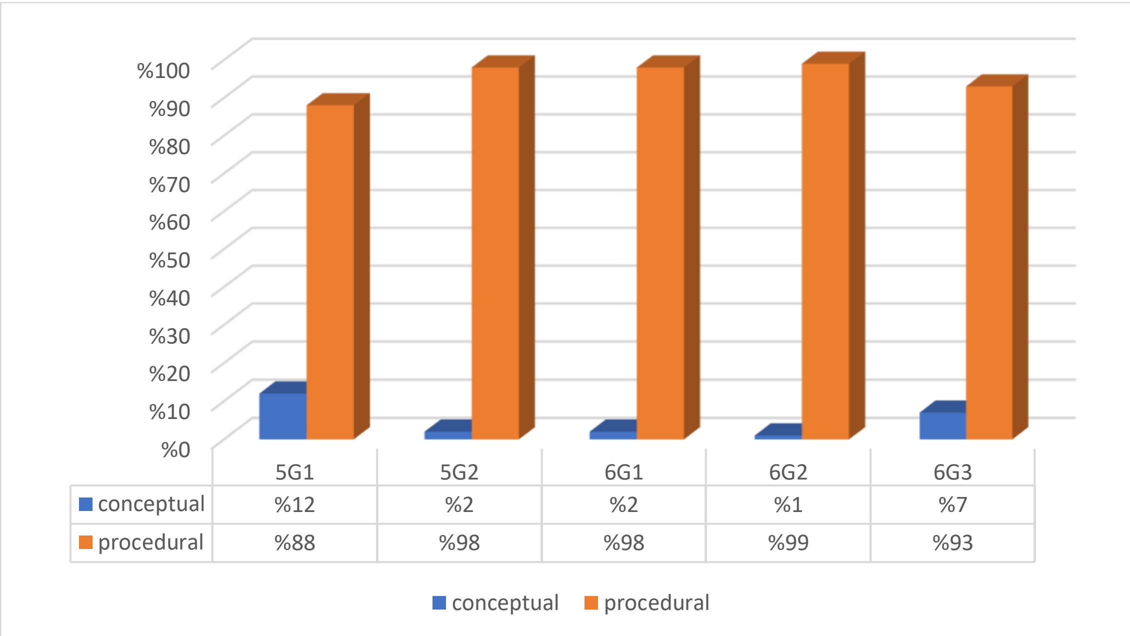
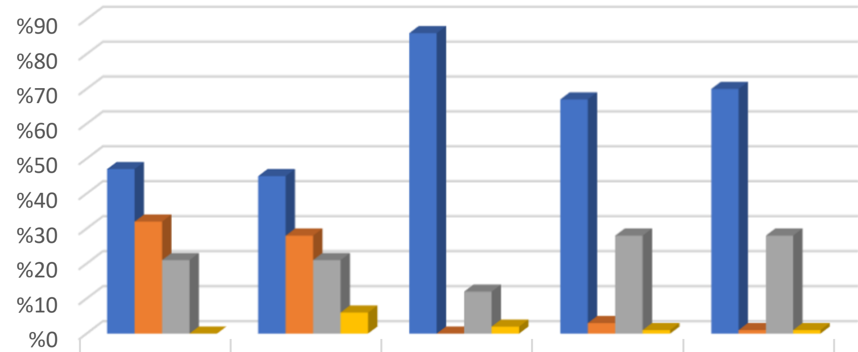


Figure 4: Distribution of Tasks in Mathematics Textbooks by Types of Knowledge

It was seen that in the examined textbooks, the tasks related to area measurement mostly necessitated procedural knowledge. It was found as a result of the examination of the textbooks that 88% of the tasks in the 5th-grade textbook with the code "5G1," 98% in the 5th-grade textbook with the code "5G2" and the 6th-grade textbook with the code "6G1," 99% in the 6th-grade textbook with the code "6G2," and 93% in the 6th-grade textbook with the code "6G3" require procedural knowledge.

3.3. Distribution of Tasks in Mathematics Textbooks by the Number of Solution Methods and Outcomes

The distribution of tasks in the five textbooks examined as a part of this study by their number of solution methods and outcomes, that is, whether they have multiple correct outcomes, a single correct outcome, multiple solution methods, and a single solution method, is given in Figure 5.



	5G1	5G2	6G1	6G2	6G3
A single correct outcome	%47	%45	%86	%67	%70
Multiple correct outcomes	%32	%28	%0	%3	%1
A single solution method	%21	%21	%12	%28	%28
Multiple solution methods	%0	%6	%2	%1	%1

Figure 5: Distribution of Tasks in Mathematics Textbooks by the Number of Solution Methods and Outcomes

It was seen that in the examined textbooks, the worked examples and activities related to area measurement mostly had a single solution method. It was also found that the worked example, activity, exercise, and unit evaluation tasks in mathematics textbooks mostly had a single outcome. It was found as a result of the examination of the textbooks that 21% of the worked example and activity tasks in the 5th-grade textbooks with the codes “5G1” and “5G2,” 12% in the 6th-grade textbook with the code “6G1,” and 28% in the 6th-grade textbooks with the codes “6G2” and “6G3” have one solution method. By and large, in the examined mathematics textbooks, there are very few worked tasks and activities with multiple solution methods; in fact, in the 5th-grade textbook with the code “5G1,” no worked example or activity with multiple solution methods was found. It was found as a result of the examination of the textbooks that 47% of the worked example, activity, exercise, and unit evaluation tasks in the 5th-grade textbook with the code “5G1,” 45% in the 5th-grade textbook with the code “5G2,” 86% in the 6th-grade textbook with the code “6G1,” 67% in the 6th-grade textbook with the code “6G2,” and 70% in the 6th-grade textbook with the code “6G3” have a single outcome. By and large, in the examined mathematics textbooks, there are fewer worked examples, activity, exercises, and unit evaluation tasks with multiple outcomes. In the 6th-grade textbook with the code “6G1,” no worked example, activity, exercise, and unit evaluation task with multiple outcomes was found.

3.4. Distribution of Tasks in Mathematics Textbooks by Contextual Features

The distribution of tasks in the five textbooks examined as a part of this study by their contextual features, that is, whether they are intra-mathematical, or realistic or authentic application problems, is given in Figure 6.

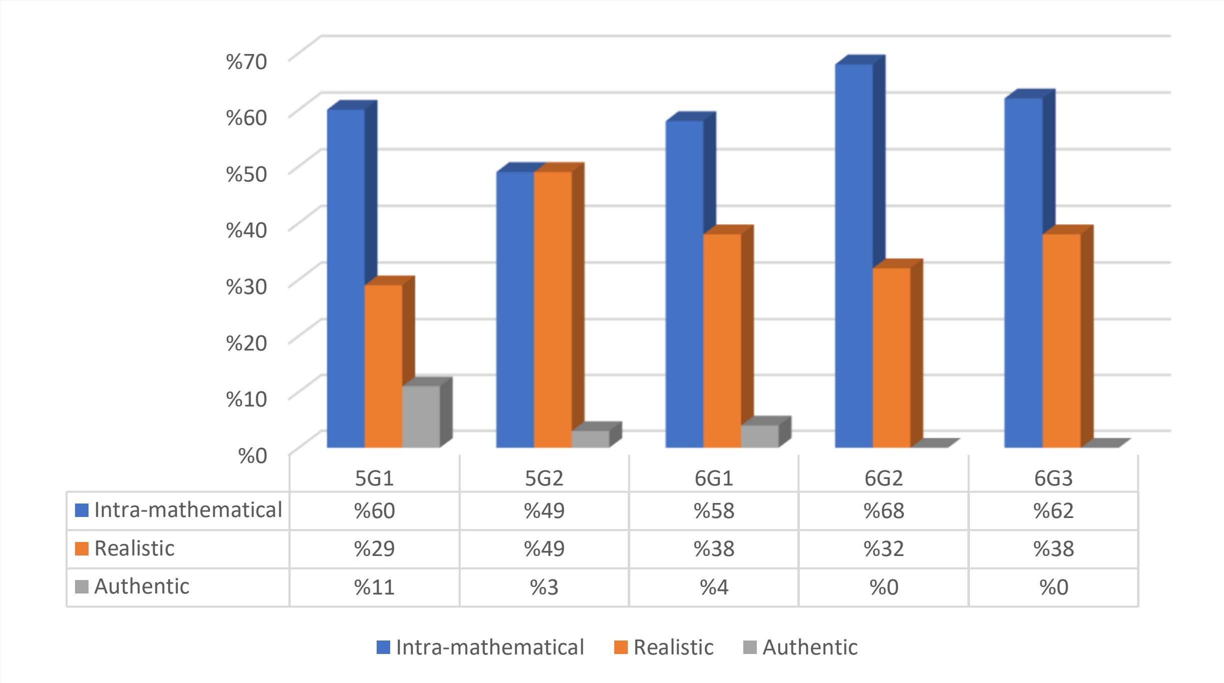


Figure 6: Distribution of Tasks in Mathematics Textbooks by Contextual Features

As a result of the examination of the five different 5th- and 6th-grade mathematics textbooks, it was found that by their conceptual features, most of the worked example, activity, exercise, and unit evaluation tasks were intra-mathematical. The intra-mathematical tasks were followed by realistic and authentic application problems, respectively. It was found as a result of the examination of the textbooks that excluding instructions, 60% of the tasks in the 5th-grade textbook with the code "5G1," 49% of the tasks in the 5th-grade textbook with the code "5G2," 58% of the tasks in the 6th-grade textbook with the code "6G1," 68% of the tasks in the 6th-grade textbook with the code "6G2," and 62% of the tasks in the 6th-grade textbook with the code "6G3" were intra-mathematical by their contextual feature. Differently from the other examined textbooks, the shares of the intra-mathematical and realistic application problems were found to be 49% and 51%, respectively, which points to a roughly equal distribution. By and large, in the examined mathematics textbooks, the number authentic application problems was found to be few, and in the 6th-grade textbooks with the codes "6G2" and "6G3," no authentic application problem was found.

4. Discussion, Conclusion, And Recommendations

In this study, in order to discover the opportunities offered by middle school mathematics textbooks to learn area measurement, the instructions, worked examples, exercises, activities, and unit evaluation questions in these textbooks were evaluated within a framework developed based on the literature.

Before the formula of "$length \times width$" is introduced to students, concepts related to area measurement such as partitioning, covering, and array structure should be taught (Battista, 2004; Outhred & Mitchelmore, 2000; Sarama & Clements, 2009). In cases where textbooks only provide students with the formula without teaching the said aspects of area measurement, teachers overlook these aspects while designing course content, which consequently undermines students' experiences and meaningful learning of area measurement. This may limit the opportunities to learn that enable students to make sense of and work on concepts related to area measurement. Therefore, in textbooks, the order and timing in which tasks related to area measurement are presented are important. It was seen that in the mathematics textbooks examined as a part of this study, the order and timing of the tasks related to area measurement were suitable for the learning outcomes of the middle school mathematics curriculum. While in some of the textbooks the formula "$length \times width$" was presented with no background information, in some textbooks, concepts related to area measurement such as partitioning, covering, and array structure was introduced before, the formula was presented. Therefore, it can be said that in terms of the timing and order of the tasks, the examined textbooks provide students with opportunities to learn that will facilitate their comprehension of the topic of area measurement as well as concepts related to it.

The findings of the study show that the content in Turkish middle school mathematics textbooks (instructions, worked examples, exercises, activities, and unit evaluation questions) mostly emphasize procedural information. This finding supports the conclusions of the study carried out by Smith et al. (2016), in which they examined American mathematics textbooks in terms of tasks related to area measurement and concluded that over 87% of the examined tasks emphasized procedural knowledge. In their study comparing Korean and American textbooks in terms of tasks related to area measurement, Hong et al. (2018) concluded that the area measurement tasks in the textbooks they examined drew more on procedural knowledge. As both countries' textbooks present the formula right after the definition of the concept of area, many tasks in these books are procedural. The textbooks from both countries were found to provide few opportunities to learn that enable the elimination of misconceptions about covering, array structure, and linking array structure to area formula. The findings of the study showed that American and Korean mathematics textbooks present procedural contents such as partitioning areas without array structure or counting square units without array structure that do not provide opportunities for students to discover the array structure or improve their skills of covering a region completely. The finding of this study that the middle school mathematics textbooks are procedural knowledge-intensive support the conclusion of Hong et al. (2018). In a study comparing the Trends in International Mathematics and Science Study achievement of 4th- and 8th-grade students in the United States between 1995 and 2003 with the achievement of students from over 25 different countries in the same period, it was found that although progress was observed in terms of achievement, students' overall performance was found to be poorer compared to students from other countries (Gonzales et al., 2004). According to Snider (2004), the reason for the decline in achievement is that although primary school textbooks that cover a large number of subjects, they cultivate shallow learning that does not enable students to develop their basic skills. The researcher also emphasized that education in the USA usually revolves around textbooks with procedural knowledge. Kridler (2012), on the other hand, emphasized that when student achievement trends are examined, procedural knowledge does not seem to positively contribute to student achievement. The opinions of these two researchers show that textbooks that lack in terms of content variety and the adoption of teaching methods drawing on procedural knowledge do not contribute to student achievement. Linking procedural to conceptual knowledge enables students to recognize real-life problems and adjust their knowledge in the face of new problems or challenges, make fewer errors, discover computational shortcuts, and learn facts and procedures easier (Baroody et al., 2007). Procedural and conceptual knowledge cannot flourish independently from one another (Baroody et al., 2007; Kilpartick et al., 2001; Lin et al., 2013). Therefore, linking the two is important in terms of enabling students to find solutions to various different problem situations. However, it was found in this study that the examined contents of the middle school mathematics textbooks emphasize procedural knowledge more than conceptual knowledge. This finding means that conceptual knowledge, which helps students to determine the appropriate procedures for solving the problems they encounter and apply these procedures to different problem situations (Zhao, 2018), is underutilized in the textbooks. Therefore, it is thought that due to the imbalance of types of knowledge they contain, the textbooks examined as a part of this study provide few opportunities to learn. It is recommended that in order to enrich education in terms of opportunities to learn and eliminate the disadvantages caused by learning based on only one type of knowledge, the same importance should be attached to two types of knowledge and contents that encourage students to link these two types of knowledge should be utilized.

It was seen that most of the activities and worked examples in the middle school mathematics textbooks had one solution method and one outcome. It was found that (excluding instructions), most tasks in the textbooks had one outcome. In the study conducted by Bingölbali (2020) to determine to what extent 6th, 7th, and 8th-grade mathematics textbooks offer questions with multiple outcomes and multiple solution methods, it was also found that 8% of the examined questions had multiple outcomes while 92% had only one outcome. Furthermore, it was found that only 6.2% of all examined worked questions could be solved through multiple methods. This finding in Bingölbali (2020) shows similarity with the finding of our study that the majority of the tasks in the examined textbooks have a single solution method and a single outcome. It was observed that tasks with multiple solution methods enhanced the quality of mathematics lessons as they encouraged students to discuss their opinions, compare different solution methods, examine the relationships between different concepts, and improve their knowledge by shifting between different representations (Leikin & Levav-Waynberg, 2008; Yackel & Cobb, 1996). It was found in our study that the number of questions in middle school mathematics textbooks that can be solved through multiple different methods is extremely few. This means that the contents of these textbooks are insufficient in terms of comparison between different solution methods and shifts between different representations, and therefore, they do not provide enough learning opportunities for students to utilize in improving themselves in these respects. It is held that textbooks, which are the primary teaching resources, should be enriched with tasks with multiple solution methods in order to ensure that they improve students' problem-solving skills, creativity, and mental flexibility, reinforce their mathematical understanding, reasoning, and thinking, and strengthen their network of ideas.

It was found that the contents related to area measurement (worked examples, exercises, activities, and unit evaluation questions) in middle school mathematics textbooks were intra-mathematical in terms of their contextual features. Similarly, in a study carried out by Glasnovic-Gracin (2018) to examine the tasks in the most commonly used Croatian mathematics textbooks in the 6th, 7th, and 8th grades, it was found that most of the tasks were intra-mathematical. In a study carried out by Cannon (2021) to examine the opportunities to develop concept images of polygons in middle school (6th, 7th, and 8th grades) mathematics textbooks, all of the three middle school mathematics textbook series that were examined were found to have more purely mathematical tasks than those related to the real world. On the other hand, Sullivan et al. (2012) stated that contextualized tasks have great potential to attract students' attention and show that they can help them make sense of the world of mathematics. It can be held that thanks to such contents, textbooks can offer better opportunities to learn. However, the finding of this study that the tasks in the examined textbooks were mostly intra-mathematical points to the fact that the number of contextual learning opportunities offered by middle school mathematics textbooks is fewer. While real-life problems have such an important role in the development of various skills of students, it is of course important to include more such content in the textbooks that represent the respective curriculum. Therefore, it is recommended to enrich textbooks in a way to include authentic and realistic application problems in addition to intra-mathematical problems.

In the continuation of this study, the subjects related to the area in middle school textbooks can also be analyzed according to this framework. It is recommended to analyze the content related to area measurement in mathematics textbooks not only at the middle school level but also at the primary school level.

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